

期貨在金融機構風險 規避上之應用

林 筠 李正福 Morgan J. Lynge, Jr.

中文摘要近年來金融機構在法令管制逐年放寬後，除了面臨更大的利率風險外，同時在資金來源與用途上也增加了更多的不確定性。銀行傳統的缺口管理法已難以應付金融環境轉變後產生的多重風險。一九七〇年代金融期貨市場的成立，為銀行提供了更多的避險工具。本文乃應用投資組合理論推導出銀行面臨利率、存款來源、與放款承諾實借額度不確定性等多重風險時，如何利用金融期貨採行更有效率的總體避險策略。

林 筠為國立台灣大學副教授

李正福為美國羅格斯大學教授

Morgan J. Lynge, Jr.為美國伊利諾大學教授

HEDGING FINANCIAL INSTITUTION GAPS WITH FUTURES

Yun Lin

National Taiwan University

Cheng F. Lee

Rutgers University at New Brunswick

Morgan J. Lynge, Jr.

University of Illinois at Urbana-Champaign

Abstract

The inception of financial futures markets gives financial intermediaries the opportunity to manage their exposure to interest rate risk, even in the face of uncertainty about future period deposit flows and loan takedown. This paper devises alternative strategies for hedging the overall risk exposure of a bank. In particular, we investigate how bank management can implement a more effective multiple crosshedging strategy to manage the interest rate risk and the quantity risk. The optimal macrohedge position can be obtained by estimating coefficients from a system of equations derived from the utility maximization problem. Then, the optimal hedge ratio for each hedging instrument is calculated from the linear combination of the coefficient estimates.

1. Introduction

High and volatile interest rates compounded with contemporary rising

inflation have not only exposed financial intermediaries to interest rate risk but have also resulted in dramatic changes in the composition of balance sheets of financial intermediaries. The conventional source of funds—low cost core deposits—has been replaced with high cost and variable rate funding vehicles. In the face of the loss of low cost deposits and greater interest rate uncertainty, financial intermediaries may choose to adjust their asset/liability strategies to "immunize" their balance sheets. Traditional Gap management techniques are commonly recommended to accomplish this immunization. However, in addition to practical problems involved in altering a balance sheet gap, gap management models usually do not consider quantity risk resulting from uncertain deposit flows, uncertain loan commitment takedowns, or loan prepayments. In general, the amount and timing of these quantity risks are dependent upon the spread between the contractual rates and prevailing market rates. Should rates rise, funds in demand and time deposit accounts may be withdrawn from the bank to be deposited in higher earning accounts. On the other hand, should rates fall, the proportion of fixed rate loan commitment takedown may be lower, while the possibility of loan prepayments may be higher. These factors introduce a new dimension of risk faced by the bank and should not be ignored in the bank decision-making process. Another shortcoming of the traditional gap models is that they represent interest rate risk exposure at a point in time and not address the evolution of risk exposure over time. It is desirable to account for the change in the composition of the bank balance sheet during the planning period. Consequently, a more effective hedge strategy should be devised to incorporate the influences of quantity risk as well as the dynamic nature of the gap position during the hedge period.

This paper investigates alternative strategies for hedging the overall risk exposure of a bank. First, we examine the traditional asset-liability management approach by using only cash market instrument. Second, we illustrate how more cost-efficient hedging strategies can be performed by bank management using futures contracts. Finally, we develop a two-period macrohedge model in an attempt to deal with the difficulties involved in the previous hedging strategies. In particular, we investigate how bank management can implement a more effective multiple crosshedging strategy

to manage the multiple risk exposure on both sides of the bank balance sheet.

2. Gap Management

Net interest income (NII) is the most commonly used focus for the interest sensitive portion of bank profitability. NII can be formulated as:

$$NII = r_n A_n + r_s A_s - i_d D - i_s B \quad (1)$$

where

A_s = the quantity of rate-sensitive assets,

A_n = the quantity of non-rate-sensitive assets,

D = core deposits which are considered to be non-rate-sensitive,

B = the purchase or sale of funds which are rate-sensitive liabilities,

r_s = the rate earned on interest sensitive assets,

r_n = the rate earned on the non-interest-sensitive asset ,

i_s = the rate paid out on the interest sensitive liabilities, and

i_d = the core deposit interest rate set by the bank.

Rearranging (1) we have

$$NII = (A_s - B)r_s + B(r_s - i_s) + r_n A_n - i_d D \quad (2)$$

The dollar GAP is the difference between the dollar amount of rate-sensitive assets (RSA) and that of rate-sensitive liabilities (RSL), i.e., $A_s - B$ in (2). Accordingly, the change in NII of a bank depends upon the relative magnitudes of the impacts of changes in market rates on RSA and RSL, or:

$$\Delta NII = \$GAP(\Delta r_s) + B(\Delta r_s - \Delta i_s) \quad (3)$$

where Δr represents the change in money market rates of interest. Basic gap management models often ignores the sensitive spread risk (where the

sensitive spread is defined as $r_s - i_s$), by assuming that $r_s - i_s$ is a constant so that

$$\Delta NII = \$GAP \times \Delta r_s \quad (4)$$

For an increase in the interest rate with $r > 0$, the NII of a bank varies directly with the sign and the size of the GAP of the bank, and vice versa.

Accordingly, a bank may choose from two gap management strategies, an active strategy and a passive strategy. If interest rates are expected to rise, for example, aggressive gap managers would set a positive gap to profit from future interest rate changes. It should be noted that the success of an active gap management strategy depends on whether the bank can predict the direction of interest rate changes accurately and adjust the gap precisely. If interest rates change in the wrong direction, the greater the gap, the greater is the bank's risk exposure for its NII. Therefore, the bank may pursue a passive gap management strategy and immunize its NII against interest rate changes. It can be achieved by restructuring its balance sheet and set the fund gap to zero.

3. The Use of Financial Futures in Bank Risk and Profitability Management

The optimal GAP position in (4) presumes that the bank can adjust the gap position in the cash market efficiently. Due to practical problems and regulatory constraints, this presumption has been criticized as unrealistic in practice. The availability of financial futures markets provides financial intermediaries with a flexible vehicle to manage their interest rate risk. By taking an appropriate position in the financial futures market financial intermediaries may immunize against the interest rate risk.

Although the potential benefits of the use of futures contracts have been confirmed in the hedging literature, banks have been using the futures markets almost exclusively to microhedge. Futures contracts are usually utilized by banks to hedge a particular operation, either to protect the value of a fixed income security, to lock in borrowing costs, or to lock in a return on pending investments (Booth et. al. 1984). However, it has been pointed out by Kolb, Timme and Gay (1984) and Jacobs (1982), among others, that a discretionary microhedging strategy employed by the financial intermediary may eventually increase overall risk by reducing the "natural

hedge". If banks are to use the futures market for a macrohedge, the futures position needed for various activities within the bank can be netted out and the transaction costs can be lowered. Therefore, the most effective way a financial intermediary can use the futures market is to implement a macrohedging strategy to hedge the total risk exposure of the bank.

Assume the bank has access to futures contracts to manage the interest rate risk of its NII. Then, the NII of the hedged portfolio becomes :

$$NII = GAPr_s + B(r_s - i_s) + r_n A_n - i_d D + y(r_{y2} - r_{y1}) \quad (5)$$

where y denotes the dollar amount of interest rate futures contract to sell ($y > 0$) or buy ($y < 0$) in the futures market, and r_{yt} denotes the implicit futures interest rate at time t .¹ The change in bank NII becomes

$$\Delta NII = GAP\Delta r_s + B\Delta(r_s - i_s) + y\Delta r_y \quad (6)$$

Again, assuming a constant sensitive spread ($\Delta r_s - \Delta i_s$), and a constant basis ($\Delta r_s = \Delta r_y$) a "perfect hedge" futures position (which insulates NII from interest rate risk) can be derived as

$$y^* = -GAP \quad (7)$$

That is, the bank should fully hedge its positive (negative) GAP position by going long (short) futures contracts. Note that the perfect hedge result can be obtained only under several restrictive assumptions. One unrealistic assumption is that interest rate changes for assets and liabilities are of the same magnitude for all maturities. Therefore, it is only the interest rate level risk that matters. If we relax this assumption and allow a stochastic sensitive spread risk, the strategy in (7) will not ensure a perfect hedged result. Dew (1981) presents a strategy to deal with a non constant sensitive spread risk.

The use of futures contracts can also be incorporated into a portfolio selection model. Portfolio approaches to hedging generally date back to Johnson (1960) and Stein (1961). However, almost all of the portfolio models focus only on hedging a single cash instrument. None of the portfolio models explicitly considers the impact of the gap position of the bank on the

¹ $(1 - r_{yt}) \times 100$ denotes the interest rate futures index price at time t .

optimal hedging decision. If the goal of bank management is to maximize the expected utility of NII by selecting the optimal gap position and futures position, then under the mean-variance framework we have the following objective function:

$$\max_{y, GAP} EU(NII) = E(NII) - 1/2[\Gamma var(NII)] \quad (8)$$

where NII was defined in (7), and y denotes the futures position. Solving the first order conditions of the maximization problem (8) the optimal futures market position and optimal cash market gap position are obtained as:

$$y^* = \frac{1}{\Gamma(1-\rho^2)} \left[\frac{E(\tilde{r}_{y2} - r_{y1})}{Var(\tilde{r}_s)} - \frac{\rho^2 E(\tilde{r}_s)}{Cov(\tilde{r}_s, \tilde{r}_y)} \right] - \frac{B}{1-\rho^2} [\beta_{\delta_s/r_y} - \beta_{\delta_s/r_s} \beta_{r_s/r_y}] \quad (9)$$

and

$$GAP^* = \frac{1}{\Gamma(1-\rho^2)} \left[\frac{E(\tilde{r}_s)}{Var(\tilde{r}_s)} - \frac{\rho^2 E(\tilde{r}_{y2} - r_{y1})}{Cov(\tilde{r}_s, \tilde{r}_y)} \right] - \frac{B}{1-\rho^2} [\beta_{\delta_s/r_s} - \beta_{\delta_s/r_y} \beta_{r_y/r_s}] \quad (10)$$

where ρ = the correlation coefficient between r_s and r_y , $\beta_{x/y}$ = the regression coefficient from regressing x against y , and δ_s denotes the sensitive spread, $\tilde{r}_s - \tilde{i}_s$.

Although the optimal gap position is mathematically attainable, in practice it is questionable whether the bank has sufficient control over its gap position to make frequent adjustments. If this is the case, then trading in the futures market would provide the bank with a flexible means to manage its interest rate risk at minimum expense. In this case the optimal futures market position is determined by

$$y^* = \frac{E(\tilde{r}_{y2} - r_{y1})}{\Gamma Var(\tilde{r}_y)} - GAP \beta_{r_s/r_y} - B \beta_{\delta_s/r_y} \quad (11)$$

The first term in (11) represents the pure speculation position and the last two terms represent the pure hedge position. If the futures interest rate

is expected to rise, the bank should take a short futures position from a purely speculative point of view and vice versa. The pure hedge position is also interpreted as the risk-minimization hedging strategy. In general, cash market interest rates move in the same direction as futures market interest rates, thus $\beta_{r_x/r_y} > 0$. Since a negative GAP position leaves the bank vulnerable to interest rate increases, from the second term in (11) we know that the bank should go short in the futures market. The opposite holds for the hedge of a positive gap position. Furthermore, if the bank is concerned with the sensitive spread risk, the third term in (11) captures the impact of this additional risk on the optimal hedging decision. If the randomness of the sensitive spread varies directly with the futures market rate, the existence of the sensitive spread risk would increase (decrease) the size of overall long (short) futures position. Since the optimal futures position consists of three terms, the bank should take either a long or a short futures position depending upon the relative effects of these three determinants.

4. Hedging Interest Rate Risk and Quantity Risk

This section develops a two-period macrohedge model that integrates the theory of hedging and theory of the banking firm to examine optimal hedging strategies as well as optimal cash market positions for a financial intermediary facing uncertain core deposit flows. The loan commitment takedown risk will be introduced in the following section. Koppenhaver (1985), Morgan and Smith (1986), and Morgan, Shome, and Smith (1988) also develop models incorporating uncertainty about deposit supply, loan takedown, or both.

Following the conventional portfolio selection approach, we assume that the utility function of bank management is characterized by constant absolute risk aversion and that random variables are jointly normally distributed. These simplifications allow us to analyze the sophisticated macrohedging strategy in the mean-variance framework. The objective of bank management is to maximize the expected utility of NII at the end of the second period by selecting the optimal macrohedge position and/or its cash market position. In order to capture the impact of the evolution of a bank's gap position on the optimal decision, we assume that the bank supplies loan

commitments at an unknown rate r_{s2} . The loans, assumed to be fully taken, have one period to maturity and no default risk.

At the beginning of the first period, the bank must determine its supply of loan commitments and the futures market position to hedge the uncertain loan return as well as the uncertain funding cost. It is assumed that the bank can borrow up to an exogenously determined amount in the form of deposits at a constant rate, i_d . Additional funds can be obtained from an established short term market at a constant but unknown borrowing rate, i_{s2} . At the beginning of period two, the available volume of deposits is revealed and the bank finances its loan demand partly with low-cost deposits and partly with high cost purchased money. All RSAs and RSLs are to mature or to be repriced at the end of period one.

At the beginning of period one, the decision-making problem is to maximize the expected utility of NII at the end of period two (NII_2) subject to the refinancing constraint, i.e.,

$$\begin{aligned} \max_{y, A_2} \phi &= EU(NII_2) & (12) \\ \text{st. } B_2 &= A_2 - (D_2 - D_1) + B_1 \\ &- (r_{s1}A_s + r_nA_n - i_dD_1 - i_{s1}B_1) \end{aligned}$$

where A_2 denotes the supply of loan commitments to be taken down in period two. The net interest income at the end of second period, which is the "target account", can be expressed as :

$$NII_2 = r_nA_n + \tilde{r}_{s2}A_s + \tilde{r}_{s2}A_2 - i_dD_2 - \tilde{i}_{s2}B_2 + yR_2(\tilde{r}_{y2} - r_{y1}) \quad (13)$$

where

$R_2 =$ one plus the risk-free rate prevailing in the second period.

The balance sheet of the bank in the beginning of the hedge period is characterized as :

$$A_s + A_n = D_1 + B_1$$

Since the model concerns the quantity risk associated with the uncertain supply of core deposits in the second period, the randomness of the deposit flow is expressed as :

$$D_2 = D_1 + \tilde{\epsilon}$$

By rearranging (13) and substituting the refinancing constraint into NII_2 ,

$$\begin{aligned} NII_2 = & (\tilde{r}_{s2} - \tilde{i}_{s2})(A_2 + A_s) + (\tilde{i}_{s2} - i_d)\Delta D \\ & + \tilde{i}_{s2}[GAP(r_{s1} - r_n + 1) + B_1(r_{s1} - i_{s1}) + D_1(r_n - i_d)] \\ & + r_n A_n - i_d D_1 + y R_2(\tilde{r}_{y2} - r_{y1}) \end{aligned} \quad (14)$$

Assuming the bank determines its loan supply and the futures hedging position simultaneously, then solving the first order conditions for (14) yields the optimal futures position

$$\begin{aligned} R_2 y^* = & \frac{1}{\Gamma(1 - \rho^2)} \left[\frac{E(\tilde{r}_{y2} - r_{y1})}{Var(\tilde{r}_{y2})} - \frac{\rho^2 E(\tilde{\delta}_s)}{Cov(\tilde{r}_{y2}, \tilde{\delta}_s)} \right] \\ & - \frac{1}{1 - \rho^2} [(\beta_{\Delta c/r_y} - \beta_{\Delta c/\delta_s} \beta_{\delta_s/r_y}) + K(\beta_{i_s/r_y} - \beta_{i_s/\delta_s} \beta_{\delta_s/r_y})] \end{aligned} \quad (15)$$

and the optimal loan commitment supply

$$\begin{aligned} A_2^* = & \frac{1}{\Gamma(1 - \rho^2)} \left[\frac{E(\tilde{\delta}_s)}{Var(\tilde{\delta}_s)} - \frac{\rho^2 E(\tilde{r}_{y2} - r_{y1})}{Cov(\tilde{r}_{y2}, \tilde{\delta}_s)} \right] \\ & - \frac{1}{1 - \rho^2} [A_s(1 - \rho^2) + (\beta_{\Delta c/\delta_s} - \beta_{\Delta c/r_y} \beta_{r_y/\delta_s}) \\ & + K(\beta_{i_s/\delta_s} - \beta_{i_s/r_y} \beta_{r_y/\delta_s})] \end{aligned} \quad (16)$$

where

Δc = addition cost (or premium) due to random deposit flows

$$= (\tilde{i}_{s1} - i_d)\Delta D,$$

$K = GAP(r_{s1} - r_n + 1) + B_1(r_{s1} - i_{s1}) + D_1(r_n - i_d)$, and

ρ = the correlation coefficient between δ_s and r_y .

On the other hand, if the cash market position of the bank is predetermined or if the bank wants to separate its normal business operations from

the hedging program, the optimal futures market position is :

$$\begin{aligned}
 R_2 y^* &= \frac{E(\tilde{r}_{y2} - r_{y1})}{\Gamma Var(\tilde{r}_{y2})} - (A_2 + A_s) \frac{Cov(\tilde{\delta}_s, \tilde{r}_{y2})}{Var(\tilde{r}_{y2})} \\
 &\quad - \frac{Cov[(\tilde{i}_{s2} - i_d)\Delta D, \tilde{r}_{y2}]}{Var(\tilde{r}_{y2})} \\
 &\quad - [GAP(r_{s1} - r_n + 1) + B_1(r_{s1} - i_{s1}) + D_1(r_n - i_d)] \frac{Cov(\tilde{i}_{s2}, \tilde{r}_{y2})}{Var(\tilde{r}_{y2})} \\
 &= \frac{E(\tilde{r}_{y2} - r_{y1})}{\Gamma Var(\tilde{r}_{y2})} - (A_2 + A_s)\beta_{\delta_s/r_y} - \beta_{\Delta c/r_y} - K\beta_{i_s/r_y}
 \end{aligned} \tag{17}$$

In (17) a more effective and cost efficient macrohedging strategy, as compared to microhedges, is suggested which simultaneously considers the interest rate risks incurred on both sides of the bank balance sheet. The optimal futures position recommended in (17) can be decomposed into two components: the pure speculation component and the pure hedge component. The first term in (17) is the speculation term of an optimal hedging strategy. This term reflects the effects of the expected return in the futures market on the bank's optimal hedging decision. Since

$$\frac{\partial y^*}{\partial E(\tilde{r}_{y2} - r_{y1})} = \frac{1}{\Gamma Var(\tilde{r}_{y2})} \times \frac{1}{R_2} > 0$$

with an expected decline in the futures price (i.e., an increase in the futures interest rate), the bank should increase its overall short hedge position or reduce its overall long hedge position. In addition to the speculation component, the remaining three terms in (17) refer to the pure hedge components of the optimal hedging strategy. The second term in (17) captures the impact of the interest rate spread risk on the optimal futures position. If the rate sensitive spread is positively correlated with the futures interest rate movement, the bank should take a long position in the financial futures market to hedge the interest spread earned on its rate sensitive assets. Since

$$\begin{aligned}
 \frac{\partial y^*}{\partial (A_2 + A_s)} &= - \frac{Cov(\tilde{r}_{s2} - \tilde{i}_{s2}, \tilde{r}_{y2})}{Var(\tilde{r}_{y2})} \times \frac{1}{R_2} \\
 &= -\beta_{\delta_s/r_y} \times \frac{1}{R_2}
 \end{aligned}$$

the greater the quantity of rate sensitive assets the more likely the optimal futures position will be a long position, providing that $cov(\tilde{r}_{s2} - \tilde{i}_{s2}, \tilde{r}_{y2}) > 0$. The third term in (17) illustrates the influence of uncertain funding costs, which in turn depends upon the quantity risk involved in the deposit supply and interest rate risk involved in the unknown borrowing rate in period two. If deposit supply is independent of interest rate uncertainty, then the third term in (17) can be rewritten as

$$-E(\Delta D) \times \frac{Cov(\tilde{i}_{s2} - i_d, \tilde{r}_{y2})}{Var(\tilde{r}_{y2})} \times \frac{1}{R_2}$$

It is likely that borrowing rates will move in the same direction as futures interest rates so that $cov(\tilde{i}_{s2} - i_d, \tilde{r}_{y2}) > 0$. If it is the case, the bank should take a short position in the financial futures market in order to hedge an expected withdrawal of deposits, i.e., $E(\Delta D) < 0$. The loss of low cost deposits increases the necessity for the bank to raise funds at higher money market rates, so by going short in the financial futures market part of the increase in the funding costs can be offset by the gain in the futures market.

The last term in (17) considers the influences of the gap position of the bank balance sheet and the interest level risk on the optimal futures position. From the discussion in the previous section, we know that a negative gap leaves the bank vulnerable to interest rate increases. If the interest rates in the cash market and in the futures market fluctuate in the same direction, the sign of the third term will depend upon the sign of GAP (assuming that the bank is able to earn a positive spread). In other words, the bank should increase its overall short futures position or reduce its overall long futures position when $GAP < 0$. On the other hand, if the GAP position of the bank is positive, the bank should reduce its overall short futures position or increase its overall long futures position.

In sum, since the risks exposed to the bank arise from a variety of sources, the optimal hedging strategy should incorporate all of these factors. The direction and the size of the net futures position depends upon the relative magnitude of sometimes conflicting impacts. Generally speaking, an expected increase in the futures interest rate, a decrease in the second period rate sensitive assets, an expected deposit withdrawal, or a negative gap in the current bank balance sheet will result in a higher overall short futures position or a lower overall long futures position and vice versa.

The size of the optimal futures position under this macrohedging strategy is in general smaller than the sum of the absolute value of discretionary microhedge positions. Therefore, the bank should be better off in the sense that more effective hedging results can be obtained with lower transaction costs.

5. Multiple Cross Hedging and Takedown Risk

In the preceding section we develop the optimal hedging strategy for a bank facing interest rate risk as well as deposit withdrawal risk. In practice, banks not only operate in the spot lending market but also in the forward lending market. While a formal loan commitment entitles the bank to supply funds up to a maximum amount, the expected size of the loan to be taken down is usually uncertain. Thus, by making a variable-rate loan commitment the bank is actually exposing itself to both interest rate risk and quantity risk. Under our setup, the model can be modified to take the uncertain loan-takedown risk into consideration. Let

$$L_2 = \tilde{\theta} A_2$$

where $\tilde{\theta}$ denotes the unknown proportion of loan commitment to be taken down. In order to hedge against both loan return risk (due to an uncertain loan demand and uncertain loan rate) and the funding cost risk (due to an uncertain deposit supply and a stochastic borrowing rate), the bank should utilize the hedging instrument in a macrohedge manner. The optimal futures position from (14) becomes:

$$\begin{aligned} R_2 y^* = & \frac{E(\tilde{r}_{y2} - r_{y1})}{\Gamma \text{Var}(\tilde{r}_{y2})} - A_2 \frac{\text{Cov}[(\tilde{r}_{s2} - \tilde{i}_{s2})\tilde{\theta}, r_{y2}]}{\text{Var}(\tilde{r}_{y2})} \\ & - A_s \frac{\text{Cov}(\tilde{r}_{s2} - \tilde{i}_{s2}, \tilde{r}_{y2})}{\text{Var}(\tilde{r}_{y2})} - \frac{\text{Cov}[(\tilde{i}_{s2} - i_d)\Delta D, \tilde{r}_{y2}]}{\text{Var}(\tilde{r}_{y2})} \\ & - [GAP(r_{s1} - r_n + 1) + B_1(r_{s1} - i_{s1}) + D_1(r_n - i_d)] \frac{\text{Cov}(\tilde{i}_{s2}, \tilde{r}_{y2})}{\text{Var}(\tilde{r}_{y2})} \end{aligned} \quad (18)$$

Although the qualitative conclusions of last section may not change substantially with this modification, the existence of uncertain loan-takedown

does exacerbate the risk exposure of the bank. Hence, in making its macro-hedge decision, the bank should pay attention to this additional quantity risk and adjust its futures hedging position accordingly.²

The presence of multiple sources of risk makes direct hedging with single futures contracts an inadequate way to protect the bank from all risk exposures. Bank management may implement a more effective multiple crosshedging strategy to manage its overall risk. Instead of using only one interest rate futures contract, the bank may also trade other types of futures contracts, say stock index futures contracts, to hedge its overall risk. If the uncertain loan-takedowns are highly correlated with the stock market index, the bank will be better off by implementing a multiple crosshedging strategy using both interest rate futures and stock index futures. Consequently, the optimal positions to be taken in the interest rate futures market, y^* , and in the stock index futures market, x^* , can be determined by solving the following problem:

$$\begin{aligned} \max \Phi_{x,y} &= EU(NII_2) & (19) \\ \text{st. } B_2 &= A_2 - (D_2 - D_1) + B_1 - (r_{s1}A_s + r_nA_n - i_dD_1 - i_{s1}B_1) \end{aligned}$$

where

$$\begin{aligned} NII_2 &= r_nA_n + r_{s2}(A_s + A_2) - i_dD_2 - i_{s2}B_2 \\ &\quad + yR_2(\tilde{r}_{y2} - r_{y1}) - xR_2(F_2 - F_1) \end{aligned}$$

Simultaneously solving the first derivatives of Φ with respect to y and x , the optimal position to be taken in the interest rate futures market is obtained as:

$$R_2y^* = \frac{1}{\Gamma(1 - \rho^2)} \left\{ \frac{E(\tilde{r}_{y2} - r_{y1})}{Var(\Delta r_y)} + \frac{\rho^2[F_1 - E(F_2)]}{Cov(\Delta r_y, \Delta F)} \right\} \quad (20)$$

²If the loan commitment A_2 is provided at fixed markup above market rates, then the optimal futures position (18) will be revised with only minor changes. That is, the second term in (18) will be substituted with

$$A_2k * \times \frac{Cov(\tilde{\theta}, r_{y2})}{Var(\tilde{r}_{y2})}$$

where k denotes the fixed markup of the loan commitment.

$$\begin{aligned}
& -\frac{1}{1-\rho^2} [A_2(\beta_{\delta_s, \theta/\Delta r_y} - \beta_{\delta_s, \theta/\Delta F} \beta_{\Delta F/\Delta r_y}) \\
& + A_s(\beta_{\delta_s/\Delta r_y} - \beta_{\delta_s/\Delta F} \beta_{\Delta F/\Delta r_y}) \\
& + (\beta_{\Delta c/\Delta r_y} - \beta_{\Delta c/\Delta F} \beta_{\Delta F/\Delta r_y}) \\
& + K(\beta_{i_s/\Delta r_y} - \beta_{i_s/\Delta F} \beta_{\Delta F/\Delta r_y})]
\end{aligned}$$

and the optimal position to be taken in the stock futures market is

$$\begin{aligned}
R_{2x^*} = & \frac{1}{\Gamma(1-\rho^2)} \left\{ \frac{F_1 - E(F_2)}{\text{Var}(\Delta r_y)} + \frac{\rho^2 E(\tilde{r}_{y2} - r_{y1})}{\text{Cov}(\Delta r_y, \Delta F)} \right\} \quad (21) \\
& + \frac{1}{1-\rho^2} [A_2(\beta_{\delta_s, \theta/\Delta F} - \beta_{\delta_s/\Delta r_y} \beta_{\Delta r_y/\Delta F}) \\
& + A_s(\beta_{\delta_s/\Delta F} - \beta_{\delta_s/\Delta r_y} \beta_{\Delta r_y/\Delta F}) \\
& + (\beta_{\Delta c/\Delta F} - \beta_{\Delta c/\Delta r_y} \beta_{\Delta r_y/\Delta F}) \\
& + K(\beta_{i_s/\Delta F} - \beta_{i_s/\Delta r_y} \beta_{\Delta r_y/\Delta F})]
\end{aligned}$$

where ρ = the correlation coefficient between Δr_y and ΔF . It is useful to note that the pure hedge positions of the multiple macrohedging strategy can be estimated from the following multiple regressions:

$$(\tilde{r}_{s2} - \tilde{i}_{s2})\theta = \alpha_1^a + \alpha_2^a \Delta r_y + \alpha_3^a \Delta F + \tilde{\epsilon}_a \quad (22a)$$

$$(\tilde{r}_{s2} - \tilde{i}_{s2}) = \alpha_1^b + \alpha_2^b \Delta r_y + \alpha_3^b \Delta F + \tilde{\epsilon}_b \quad (22b)$$

$$(\tilde{i}_{s2} - i_d)\Delta D = \alpha_1^c + \alpha_2^c \Delta r_y + \alpha_3^c \Delta F + \tilde{\epsilon}_c \quad (22c)$$

$$\tilde{i}_{s2} = \alpha_1^d + \alpha_2^d \Delta r_y + \alpha_3^d \Delta F + \tilde{\epsilon}_d \quad (22d)$$

Then, the pure hedge positions in the interest rate futures market, h_y^* , and in the stock index futures market, h_x^* , can be determined by the linear combinations of the regression estimates

$$h_y^* = -(A_2 \alpha_2^a + A_s \alpha_2^b + \alpha_2^c + K \alpha_2^d) \frac{1}{R_2} \quad (23)$$

$$h_x^* = (A_2 \alpha_3^a + A_s \alpha_3^b + \alpha_3^c + K \alpha_3^d) \frac{1}{R_2} \quad (24)$$

As an example of this estimation technique, a simplified asset and liability structure for a hypothetical bank is used. The bank's assets are limited to two types of loans, A_s and A_n ; the liabilities are limited to core

deposits and purchased funds. Assume that the cost of deposits (i_d) is fixed at 5.25 percent. The rate at which the bank is assumed to purchase funds (i_s) is the monthly secondary market rate for one month CDs. The monthly average prime rate for business loans represents a proxy for r_s . The sources of quantity risk, θ and D , are calculated from survey and Report of Condition data compiled by the Federal Reserve Board for selected large commercial banks. Specifically, the ratio of outstanding loans made under commitments to the total of unused and used commitments serves as a proxy for the loan commitment takedown rate θ . D is taken to be the average change in demand and savings deposits for reporting banks. To hedge its overall risk exposure, the hypothetical bank decides to trade in the T-bill futures and the stock index futures market simultaneously. Assuming a one month planning horizon, data on the T-bill futures contract and the S&P 500 index futures contract (with less than three months to maturity) are gathered on the last day of each month.

Table 1 contains the simplified bank balance sheet and the initial condition faced by the hypothetical bank. Table 2 presents estimates of the coefficients in (22a) through (22d) using monthly data from September 1982 to March 1985. Utilizing the information in these two tables, the pure hedge positions in the T-bill futures market and the S&P 500 futures market can be calculated from (23) and (24) as:

$$\begin{aligned} h_y^* &= -[1,500(.1150) + 4,000(.3401) + 57.3200 \\ &\quad + (-2093.75)(.7913)] \times 10^7 \times (1/1.08) \\ &= -\$159,166,667 \end{aligned}$$

$$\begin{aligned} h_x^* &= [1,500(.0099) + 4,000(.0278) + 3.4650 \\ &\quad + (-2093.75)(.7513)] \times 10^3 \times 500 \times (1/1.08) \\ &= \$58,991,296 \end{aligned}$$

These results imply that the hypothetical bank should take a net long position in T-bill futures market and a net short position in S&P 500 index futures market in order to hedge its overall risk exposure.³ Furthermore,

³Adjustments were made to consider the contract sizes of futures contracts and the discount factor R_2 (assume to be 1.08).

the speculation components in (20) and (21) can be determined as long as the two subjective factors (the risk aversion index Γ and interest rate expectations) are specified. The magnitude of the complete optimal multiple macrohedge positions of a bank would depend upon current conditions, the interest rate expectations and the risk tolerance of the bank.

6. Summary and Conclusions

With the existence of futures markets financial intermediaries have the opportunity to manage their exposure to interest rate risk, even in the face of uncertainty about future period deposit flows and loan takedown. Using a portfolio selection framework, models were developed and optimal futures market and cash market positions were derived. The models focus on the maximization of expected utility of net interest income and, in some versions, use the GAP between rate sensitive assets and liabilities as a decision variable, an approach that is widely taken in practice. In the case of multiple sources of risk, from both deposit and loan takedown uncertainty, a multiple cross hedging strategy is derived, using both interest rate and stock index futures contracts positions.

The optimal positions derived contain terms that reflect pure speculative and pure hedge components. The values and signs of the hedging terms depend on the interrelationship among cash and futures market rates of interest, future deposit flows and takedown rates, and the degree of risk aversion of the financial intermediary.

APPENDIX

First Order Conditions for Equations (8) and (12)

The objective function in equation (8) is:

$$\max_{y, GAP} EU(NII) = E(NII) - 1/2[\Gamma var(NII)] \quad (8)$$

The first order conditions of this maximization problem are derived as :

$$E(\tilde{r}_{y2} - r_{y1}) - \Gamma[yvar(\tilde{r}_y) + GAP(\tilde{r}_s, \tilde{r}_y) + Bcov(\tilde{r}_s - \tilde{i}_s, \tilde{r}_y)] = 0 \quad (8a)$$

and

$$E(\tilde{r}_s) - \Gamma[y\text{cov}(\tilde{r}_s, \tilde{r}_y) + GAP\text{var}(\tilde{r}_s) + B\text{cov}(\tilde{r}_s - \tilde{i}_s, \tilde{r}_s)] = 0 \quad (8b)$$

Simultaneously solving (8a) and (8b), the optimal futures market position and optimal cash market gap position are obtained as in equations (9) and (10) in the text.

The first order conditions for the maximization problem using equation (12), assuming that the bank determines its loan supply and the futures hedging position simultaneously, are:

$$\begin{aligned} \frac{\partial \Phi}{\partial y} &= E(\tilde{r}_{y2} - r_{y1}) - \Gamma\{y_1 R_2 \text{Var}(\tilde{r}_{y2}) \quad (12a) \\ &\quad + (A_2 + A_s) \text{Cov}[(\tilde{r}_{s2} - \tilde{i}_{s2}), \tilde{r}_{y2}] + \text{Cov}[(\tilde{i}_{s2} - i_d) \Delta D, \tilde{r}_{y2}] \\ &\quad + [GAP(r_{s1} - r_n + 1) + B_1(r_{s1} - i_{s1}) + D_1(r_n - i_d)] \text{Cov}[\tilde{i}_{s2}, \tilde{r}_{y2}]\} \\ &= 0 \end{aligned}$$

and

$$\begin{aligned} \frac{\partial \Phi}{\partial A_2} &= E(\tilde{r}_{s2} - \tilde{i}_{s2}) - \Gamma\{(A_2 + A_s) \text{Var}(\tilde{r}_{s2} - \tilde{i}_{s2}) \quad (12b) \\ &\quad + y_1 R_2 \text{Cov}[(\tilde{r}_{s2} - \tilde{i}_{s2}), \tilde{r}_{y2}] + \text{Cov}[(\tilde{r}_{s2} - i_d) \Delta D, (\tilde{r}_{s2} - \tilde{i}_{s2})] \\ &\quad + [GAP(r_{s1} - r_n + 1) + B_1(r_{s1} - i_{s1}) + D_1(r_n - i_d)] \text{Cov}[\tilde{i}_{s2}, (\tilde{r}_{s2} - \tilde{i}_{s2})]\} \\ &= 0 \end{aligned}$$

Solving (12a) and (12b) yields the optimal futures and loan commitment supply positions shown as equations (17) and (18) in text.

TABLE 1
A. Balance Sheet
(millions of dollar)

Assets		Liabilities	
A_s	\$4,000	Demand and Savings Deposits	\$3,500
A_n	6,000	Certificates of Deposit	6,500
	\$10,000		\$10,000

B. Initial Conditions

Interest Yield (percent)	Other Variables (million)
r_{s1} 12	A_2 \$ 1,500
r_n 13	GAP -2,500
i_d 5.25	k -2,093.75
i_{s1} 10	

TABLE 2
Estimation of Eq. (22a) Through Eq. (22d)

Equation	α_2 (10^{-3})	α_3 (10^{-3})
(22a)	.1150	.0099
(22b)	.3401	.0278
(22c)	57.3200	3.4650
(22d)	.7513	.0010

References

- Booth, J. R., R. L. Smith, and R. W. Stolz. 1984. Use of Interest Rate Futures by Financial Institutions. *Journal of Bank Research* (Spring): 15-20.
- Dew, J. K. 1981. Bank Regulations for Futures Accounting. *Issues in Bank Regulation* (Spring): 16-23.
- Jacobs, R. L. 1982. Restructuring the Maturity of Regulated deposits with Treasury-Bill Futures. *The Journal of Futures Markets* 2 (Summer): 183-193.
- Johnson, L. L. 1960. The Theory of Hedging and Speculation in Commodity Futures. *Review of Economic Studies* 27: 139-151.
- Kolb, R. W., S. G. Timme, and G. D. Gay. 1984. Macro Versus Micro Futures Hedges at Commercial Banks. *The Journal of Futures Markets* 4 (Spring): 47-54.
- Koppenhaver, G. D. 1985. Bank Funding Risks, Risk Aversion, and the Choice of Futures Hedging Instrument. *Journal of Finance* 40 (Spring): 241-255.
- Morgan, G., and S. Smith. 1986. Basis Risk, Partial Takedown, and Hedging by Financial Intermediaries. *Journal of Banking and Finance* 10 (Winter): 467-490.
- Morgan, G. E, D. K. Shome, and S. D. Smith. 1988. Optimal Futures Positions for Large Banking Firms. *Journal of Finance* 43 (Spring): 175-195.
- Stein, J. L. 1961. The Simultaneous Determination of Spot and Futures Prices. *American Economic Review* 51: 1012-1025.