會計評論
第 28 期，第 51-78 頁
中華民國八十三年九月

市場報酬及會計盈餘模型之簡介

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摘要

本研究主要介紹如何用一簡單的現值模型導出市場報酬及會計盈餘間之不同模式。過去的研究所用的模式常因特異的研究問題有所變動。本研究從一步步的模型發展過程中，教導會計博士班學生及研究初學者能對不同的會計模式及模式間的比較關係有深入的了解。惟有深入了解理論模型，吾人才能有效的從事實證研究之工作。在討論模式導出過程中，本研究借用已發表的研究報告以舉例說明。

THE CHINESE ACCOUNTING REVIEW
Vol. 28 PP. 51-78
September, 1994

INVITED EDITORIAL

A Simple Framework for Modeling the Explicit Earnings-Returns Relation

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Abstract

This paper provides a step-by-step illustration on how the relationship between earnings and returns can be formally modeled. By understanding the detailed processes of the model development, this paper will help accounting Ph.D. students and researchers interested in this line of research in understanding and evaluating studies of the returns-earnings relationship. Only when a full understanding of the theoretical model is obtained, can a proper empirical research design be attained.

The paper starts from a general valuation model which leads to different earnings-returns formats corresponding to different specifications seen in previous studies. After the fundamental theoretical framework is established, several studies are discussed as examples to illustrate the use of the basic framework.

1. Introduction

In the last two decades of accounting research, few empirical phenomena have drawn as much attention from accounting research as the relationship between earnings and stock returns. In the beginning of this avenue of research (Ball and Brown, 1968), researchers focus on investigating the relation between **unexpected** earnings and **abnormal** (unexpected) returns and use correlation or nonparametric methods to investigate whether or not there is a positive relationship. Not until one and a half decades ago has the explicit relationship between earnings and returns been studied. Beaver, Clark and Wright (1979) is probably one of the first studies that investigates the impact of the magnitude of unexpected earnings on abnormal returns. Beaver, Lambert and Morse (1980, BLM, henceforth) develop an explicit regression model for the unexpected-earnings/returns relation and evaluate the value of the regression coefficient of the earnings variable. Kormendi and Lipe (1987), Collins and Kothari (1989), and Easton and Zmijewski (1989) provide further evidence on the explicit relation, i.e., the earnings response coefficient (the ERC), between unexpected earnings and (unexpected) returns.

Recently, studies start to appreciate the information value of the level of earnings in explaining the variation of returns. Concurrent with Lev's (1989) report on the disappointing low R^2 of unexpected earnings variable in explaining returns, numerous studies implement new theoretical and empirical approaches to explore the earnings-returns relation. Ohlson (1989, 1991), Ohlson and Shroff (1992) explore the underlying earnings-returns relation and suggest that the earnings level variable can serve as an 'ingenuous' benchmark explanatory variable. Easton and Harris (1991) provide empirical evidence on the importance of the earnings level variable in explaining both raw and abnormal returns. Due to the theoretical development and empirical evidence, studies start to include only the earnings level variable in the earnings-returns specification e.g., Warfield and Wild (1992). Ali and Zarowin

1992) show that earnings level variable has more information content than change of earnings when the previous period earnings have less permanent earnings.

While the theoretical and empirical evidence attests the importance of earnings level variable, recent studies continue to use the traditional unexpected earnings framework that treats the change of (or unexpected) earnings as the main earnings variable in the returns regression model, e.g. Kallapur (1994), Ahmed (1994), Ali (1994) and Dhaliwal and Reynolds (1994). The credibility of these results using the traditional unexpected earnings framework needs to be evaluated with the new earnings level framework. Research has shown that the importance of the earnings level variable is not necessarily inconsistent with the traditional unexpected earnings framework while the change of earnings variable continues to provide significant additional explanatory power beyond the earnings level variable. However, in order to evaluate the past and current accounting studies in this area, it is important for accounting researchers to understand the meanings of the earnings level variable in contrast to the earnings change variable in the earnings-returns model. This understanding can only be obtained through rigorous theoretical model development.

Many past studies have provided the theoretical model for the earnings-returns relation; however, they vary in assumptions and expressions. These variations often cause confusion for researchers who are new in this area. For example, BLM derive a model relating the change of earnings scaled by previous-period earnings to returns excluding dividends. They suggest that the coefficient is purely due to the time-series property of earnings. Ohlson (1991) establishes the relation between the earnings and the sum of price and dividends. The coefficient of the model has a theoretical value of *(1+discount rate)/discount rate* which is different from the *1/(cost of capital−growth rate)* in Miller and Modigliani (1961). Kormendi and Lipe (1987) and Collins and Kothari (1989) derive a theoretical earnings response coefficient (ERC) as one plus an earnings persistence measure. And recently, followed Easton and Harris's (1992) suggestion of including earnings level variable in the abnormal returns regression, Ali and Zarowin (1992) show that earnings level and earning change have different response coefficients depending on the time-series properties of earnings. All these research share the same theoretical origin; however, due to the difficulty in modeling the real-world complexity and different focus of the research questions, their earnings-returns models differ.

This paper has a pedagogical purpose which is to provide a simple theoretical framework to illustrate the step-by-step development of some simplified theoretical models so that an understanding can be obtained as to how different theoretical models are formed. By understanding the process of the model development, this paper will help accounting Ph.D. students and researchers interested in this line of research in understanding and evaluating studies of the returns-earnings relationship. Only when a full understanding of the theoretical model is obtained, can a proper empirical research design be attained. This paper starts from a general valuation model which leads to different earnings-returns formats corresponding to different specifications seen in previous studies. After the fundamental theoretical framework is established, several studies will be discussed as examples to illustrate the use of the basic framework.

2. The Equity Valuation Model

To develop a theoretical model, researchers should start from a model with the most general form, additional assumptions can be added later on to operationalize the model. A model with assumptions is easier to handle and from which testable hypotheses can be developed; however, added assumptions limit the reality or generalizability of the model. In applying a simple model to an empirical setting, the effect of violations of the assumptions should be born in mind when making conclusions. Often, we observe that the empirical results conflict with the theoretical predictions. One likely reason is due to the invalidity of the assumptions used in the theoretical model.

This section starts from a general equity valuation model followed by a sequence of simplified equity valuation models with added assumptions; these valuation models will then lead to specifications of the returns-earnings relation. Figure 1 shows the assumptions added to a sequence of four fundamental models: Model 1 is the most general model that relates the equity value to future dividends; Model 2 is derived after adding the isomorphic assumption of the earnings and dividends; Model 3 is with the assumptions of constant future discount rate and patterned earnings series and Model 4 assumes constant earnings and is the simplest equity valuation model. Model 4 is then used to derive the returns model in the next section.

Model 1: The General Price-Expected Dividends Model

The price of security i at time t in a world without taxes can be stated:

$$P_{it}=\sum_{k=1}^{T}\frac{\widehat{D}_{it+k}^{t}}{\prod_{\tau=1}^{k}(1+\widehat{r}_{it+\tau}^{t})}, \qquad (1)^{1}$$

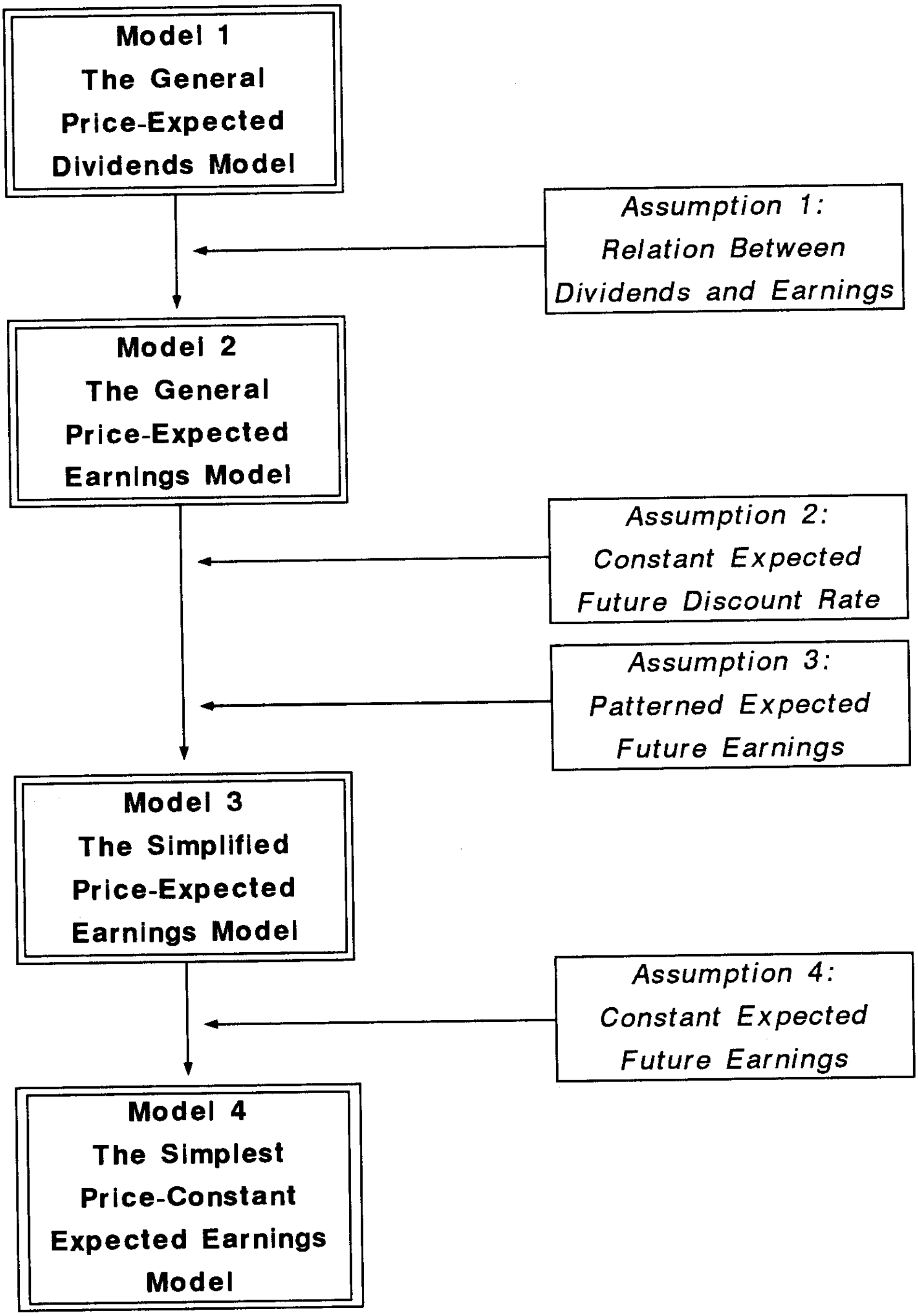
where

P_{it} = value of equity of firm i at the end of period t;

$\widehat{D}_{it+k}^{t}$ = expectation at the end of period t of dividends to be received at the end of period t+k for firm i;

[1]This model is similar to the model described in Christie(1987) and Collins and Kothari (1989) with variations on firm lives and the structure of expected discount rates.

Figure 1 The Equity Valuation Models



$\hat{r}^{t}_{it+\tau}$ =expectation at the end of period t of discount rates for the dividends to be received at the end of period $t+\tau$ for firm i;

T =number of periods remaining in the life of firm i.

If we assume that the future discount rates are known and the future cash flows constitute the only source of the admissible uncertainty in the multi-period capital asset pricing model, this model is equivalent to Fama's (1977) results (see Christie, 1987 and Collins and Kothari, 1989)

To establish a connection between the dividends and the earnings, we have to assume a relation between expected dividends and expected earnings. A general form of this assumption can be stated as:

$$\hat{D}^{t}_{it+k}=\pi_{it+k}\,\hat{E}^{t}_{it+k} \qquad \text{for all future periods } k=1...T, \tag{2}$$

where

π_{it+k} =a coefficient relates expected earnings and expected dividends for period t+k, it can be referred to as the payout rate for special cases;

$\hat{E}^{t}_{it+k}$=expectation at the end of period t for earnings at period t+k.

The simplest assumption for the relation between dividends and earnings is to assume that the expected earnings and expected dividends are equal. This can be achieved by either assuming a one hundred percent dividend payout ratio or by using the dividend irrelevance proposition in Miller and Modigliani (1961); accordingly, equation (1) can be expressed directed by expected earnings. If earnings are not paid out completely or dividends policies are relevant, then the returns from the reinvestment of the amount in excess of dividends may be modeled either explicitly or implicitly. Kallapur (1994) uses Fama and Miller's (1972) model which assumes constant payout rate to explicitly express the payout ratio and the rate of return on reinvestment of earnings in the earnings-returns relation. Collins and Kothari (1989) model the relation between dividends and earnings implicitly by linking future expected dividends to current period earnings using a nonconstant parameter, they assume:

$$\hat{D}^{t}_{it+k} = \lambda_{it+k} E_{it} \qquad \text{for all future periods } k = 1 \ldots T, \tag{3}$$

where E_{it} represents firms i's reported accounting earnings for period t and λ_{it+k} is a parameter relating the future dividends to current-period earnings. The parameter λ_{it+k} depends on the particular time series process the earnings follow as a function of the firm's investment and dividend policies (p.146).

Model 2: The General Price-Expected Earnings Model

Relating the dividends to accounting earnings is an essential step in specifying the returns and earnings relation. Ohlson (1983) demonstrates that, with certain parameter restrictions, the value of equity can alternatively be written as a discounted earnings stream, although the discount rate for the expected earnings will differ from the discount rate for the expected dividends. Accordingly, most accounting studies adopt the following equity model based on expected earnings without considering the relation between earnings and dividends. However, when the dividend payout ratio (as in Kallapur, 1994) is an issue, the payout ratio should be modeled using equation (1) as the starting point. Model 1 is restated below by substituting expected earnings into expected dividends.

$$P_{it} = \sum_{k=1}^{T} \frac{\hat{E}^{t}_{it+k}}{\prod_{\tau=1}^{k} (1 + \hat{r}^{t}_{it+\tau})} . \tag{4}$$

Model 3: The Simplified Price-Expected Earnings Model

Model 2 is a complicated nonlinear model. To operationalize the model, the time-series variations of both the expected discount rates and the expected earnings are simplified by assuming a constant expected discount rate and by assuming that earnings follow a specific time-series pattern. The time-series pattern of earnings can be specified using a general form of the integrated autoregressive and moving average (ARIMA) time-series model. An ARIMA (p,d,q) model includes a differencing order (d), the number of autoregressive terms (p) and the number of moving average terms (q). Accounting studies have used different forms of the ARIMA model to depict the time-series properties of accounting earnings. For annual earnings, studies have assumed that the annual earnings follow a random walk time-series model, a random walk with drift model (Ball and Brown, 1968), an autoregressive model with one to two lag terms of earnings (Lipe, 1986) and a moving average process (BLM, 1980; Ali and Zarowin, 1992). In accounting studies, the ARIMA models are often applied to simplify the expression of the unexunexpected earnings (the reported earnings subtract the expected earnings). The relation between the ARIMA parameters and the earnings response coefficient (the coefficient that relates the unexpected earnings to unexpected returns) has been a focus of many recent papers. I will discuss some of the ARIMA models in the example section follows the simple theoretical framework. Here, I will use an earnings growth model to illustrate the simplification of model 2.

Assume future earnings have a constant growth rate of g:

$$E_{it+k} = (1+\hat{g}_i^{\,t})^{k-1}\hat{E}_{it+1}^{\,t} + \epsilon_{it+k}, \tag{5}$$ [2]

where

$\hat{g}_i^{\,t}$ = the expected growth rate at the end of period t of firm i's future earnings;

ϵ_{it+k} = the error term that has a mean zero and a constant variance (a white noise series).

Assuming a constant discount rate, substituting (5) into equation (4) yields:

$$P_{it} = \sum_{k=1}^{T} \frac{(1+\hat{g}_i^{\,t})^{k-1}\hat{E}_{it+1}^{\,t}}{(1+\hat{r}_{it+1}^{\,t})^{k}}. \tag{6}$$

To simplify the expressions, let

$\hat{g}_{it}$ $=\hat{g}_i^{\,t}$, i.e. the expected future growth rate for firm i at the end of period t;

$\hat{E}_{it}$ $=\hat{E}_{it+1}^{\,t}$, i.e. the expected period t+1 earnings for firm i at the end of period t;

$\hat{r}_{it}$ $=\hat{r}_{it+1}^{\,t}$, i.e. the expected future discount rate for firm i at the end of period t.

Model (6) can be simplified to:

$$P_{it} = \sum_{k=1}^{T} \frac{(1+\hat{g}_{it})^{k-1}\hat{E}_{it}}{(1+\hat{r}_{it})^{k}} \tag{7}$$

$$= \frac{1 - \left(\dfrac{1+\hat{g}_{it}}{1+\hat{r}_{it}}\right)^{T}}{\hat{r}_{it} - \hat{g}_{it}}\,\hat{E}_{it}.$$

With the assumption that the growth rate is smaller than the discount rate and the firm life T approaches infinity, equation (7) can be further simplified to the model described in Miller and Modigliani (1961).

$$P_{it} = \frac{1}{\hat{r}_{it} - \hat{g}_{it}}\,\hat{E}_{it}. \tag{8}$$

Model 4: The Simplest Price-Constant Expected Earnings Model

The model in equation (8) can be further simplified by assuming

[2]This model can be expressed as ARIMA(1,1,0).

that there is no growth in earnings, that is $\hat{g}_{it}=0$ and the expected earnings for all future periods are the same:

$$P_{it}=\frac{1}{\hat{r}_{it}}\hat{E}_{it}. \tag{9}$$

This model expresses equity value as a multiple of the expected earnings and is the most popular model in the returns-earnings studies due to its simplicity. This simple model will be used to derive the returns model in the following section.

3. The Returns Model - the Reduced Form of the Price Model

Christie (1987) categorizes studies using the valuation model as the 'level' studies and the studies on returns or unexpected returns as the 'returns' studies. The returns studies use the reduced form of the equity model. The returns model is more complicated than the valuation model because it involves taking variables from two points in time and it requires assumptions of the time-series behavior of the key variables. Moreover, conventional measure of the returns are in ratio form; treatments of the scaling factor of the returns (the previous end-of-period price) will affect the expressions of the returns model. In addition, the inclusion (or exclusion) of dividends often causes confusion.

This section illustrates different expressions of the returns model that can be derived from the simplest form of the model in equation (9). These various expressions provide a framework for making comparisons between accounting studies which use different expressions of the theoretical model. To simplify the discussion, I first exclude the dividends from measuring returns.

Returns Model without dividends

Prior to Ohlson (1991), Ohlson and Shroff (1992) and Easton and Harris (1992), the earnings level variable was never used to explain the variation of returns. In fact, using the formal modeling approach, we can see that earnings level variable is clearly relevant to the returns.

The Earnings Level Model

From equation (9), the returns can be expressed as:

$$\frac{P_{it}-P_{it-1}}{P_{it-1}}=\frac{\frac{\hat{E}_{it}}{\hat{r}_{it}}-P_{it-1}}{P_{it-1}}=\frac{1}{\hat{r}_{it}}\cdot\frac{\hat{E}_{it}}{P_{it-1}}-1; \tag{10}$$

note that earnings are scaled by the prior end-of-period price in equation (10).

Alternatively, equation (10) can be expressed by expending the expression of the prior-period price as expected earnings divided by expected rate of return:

$$\frac{P_{it}-P_{it-1}}{P_{it-1}}=\frac{\hat{r}_{it-1}}{\hat{r}_{it}}\cdot\frac{\hat{E}_{it}}{\hat{E}_{it-1}}-1; \tag{11}$$

in this equation, earnings are scaled by previous period earnings.

The returns equations with earnings level described above can be restated to include the change of expected earnings by adding an assumption that the expected future discount rate does not change across different time periods (i.e. $\hat{r}_t=\hat{r}_{t-1}$).

The Earnings Change Model

The model below scales earnings by prior end-of-period price.

$$\frac{P_{it}-P_{it-1}}{P_{it-1}}=\frac{\frac{\hat{E}_{it}}{\hat{r}_{it}}-P_{it-1}}{P_{it-1}}=\frac{\frac{\hat{E}_{it}}{\hat{r}_{it}}-\frac{\hat{E}_{it-1}}{\hat{r}_{it-1}}}{P_{it-1}} \tag{12}$$

$$=\frac{1}{\hat{r}_{it}}\cdot\frac{\hat{E}_{it}}{P_{it-1}}-\frac{1}{\hat{r}_{it-1}}\cdot\frac{\hat{E}_{it-1}}{P_{it-1}}$$

$$=\frac{1}{\hat{r}_{it}}\cdot\frac{\hat{E}_{it}-\hat{E}_{it-1}}{P_{it-1}}=\frac{1}{\hat{r}_{it}}\cdot\frac{\Delta\hat{E}_{it}}{P_{it-1}}.$$

The model below scales earnings by prior period expected earnings.

$$\frac{P_{it}-P_{it-1}}{P_{it-1}}=\frac{\hat{r}_{it-1}}{\hat{r}_{it}}\cdot\frac{\hat{E}_{it}}{\hat{E}_{it-1}}-1=\frac{\hat{r}_{it-1}}{\hat{r}_{it}}\cdot\frac{(\hat{E}_{it-1}+\Delta\hat{E}_{it})}{\hat{E}_{it-1}}-1 \quad (13)$$

$$=\frac{\hat{r}_{it-1}}{\hat{r}_{it}}+\frac{\hat{r}_{it-1}}{\hat{r}_{it}}\cdot\frac{\Delta\hat{E}_{it}}{\hat{E}_{it-1}}-1=\frac{\Delta\hat{E}_{it}}{\hat{E}_{it-1}}.$$

<u>The Earnings Change and Earnings Level Model</u>

Equations (10) and (12) or equations (11) and (13) can be combined by assigning k and (1-k) to equations (10) and (12) or (11) and (13), respectively:

$$\frac{P_{it}-P_{it-1}}{P_{it-1}}=k\cdot\frac{1}{\hat{r}_{it}}\cdot\frac{\hat{E}_{it}}{P_{it-1}}-k+(1-k)\cdot\frac{1}{\hat{r}_{it}}\cdot\frac{\Delta\hat{E}_{it}}{P_{it-1}}, \quad (14)$$

$$\frac{P_{it}-P_{it-1}}{P_{it-1}}=k\cdot\frac{\hat{r}_{it-1}}{\hat{r}_{it}}\cdot\frac{\hat{E}_{it}}{\hat{E}_{it-1}}-k+(1-k)\cdot\frac{\Delta\hat{E}_{it}}{\hat{E}_{it-1}}. \quad (15)$$

All the returns models described here (equations 10 through 15) are exactly the same when the expected future discount rate at the end of period t-1 equals to that at the end of period t. If we can observe the expected earnings, then using two variables in the returns regression does not make sense because they are redundant to each other. However, if we cannot observe the expected earnings, the earnings level and earnings change model is particular useful if the measurement errors contained in the empirical variables canceled out from each other.

Returns Model with Dividends

Dividends cause complications in the returns model. Most research studies include dividends in the returns measure without dealing with the dividends explicitly. Ohlson and Penman (1992) is one of the few that reports empirical results based on returns with and without dividends. Easton and Harris (1991) adopt Ohlson's (1989) analysis and state that if dividend is paid on security j at time t, then the price equation (9) should be written as.[3]

$$P_{it}+D_{it}=\rho\widehat{E}_{it}. \tag{16}$$

Easton and Harris give no explicit definitions to the value of ρ. Ohlson and Shroff (1992) define the coefficient ρ to be $R_F/(R_F\text{-}1)$ where R_F is defined as one plus the firm's cost-of-capital. To link the model in equation (16) to the model in equation (9), equation (16) is restated:

$$P_{it}+D_{it}=(\frac{1+\widehat{r}_{it}}{\widehat{r}_{it}})\widehat{E}_{it}. \tag{17}$$

The main difference between equations (9) and (17) is the inclusion of dividends in (17). Equation (17) can be transformed to equation (9) only if the expected earnings equal to the dividends in period t, or equivalently, the expected earnings are assumed to equal to the expected dividends which are based on current dividends.

$$\begin{aligned} P_{it} &= \frac{1}{\widehat{r}_{it}}\widehat{E}_{it}+\widehat{E}_{it} - D_{it} \\ &= \frac{1}{\widehat{r}_{it}}\widehat{E}_{it}. \end{aligned} \tag{18}$$

Based on the assumption that expected earnings equal current earnings (dividends), the returns models as in equations (10) through (15) with dividends added to the left-hand-side can be derived by simply adding the expected earnings in the right hand side. I will use the previous end-of-period price instead of the previous period expected earn-

[3]The error term in their model is ignored.

ings as the scaling factor and restate equations (10), (12) and (14) as:

$$\frac{P_{it}-P_{it-1}+D_{it}}{P_{it-1}}=\frac{1+\widehat{r}_{it}}{\widehat{r}_{it}}\cdot\frac{\widehat{E}_{it}}{P_{it-1}}-1, \tag{19}$$

$$\frac{P_{it}-P_{it-1}+D_{it}}{P_{it-1}}=\frac{1}{\widehat{r}_{it}}\cdot\frac{(\Delta\widehat{E}_{it})}{P_{it-1}}+\frac{\widehat{E}_{it}}{P_{it-1}}, \tag{20}$$

$$\frac{P_{it}-P_{it-1}+D_{it}}{P_{it-1}}=k\cdot\frac{1+\widehat{r}_{it}}{\widehat{r}_{it}}\cdot\frac{\widehat{E}_{it}}{P_{it-1}}-k+(1-k)\cdot\frac{1}{\widehat{r}_{it}}\cdot\frac{(\Delta\widehat{E}_{it})}{P_{it-1}}. \tag{21}$$

The Unexpected (Abnormal) Returns Model

Starting from Ball and Brown (1968), accounting studies have been concentrated on evaluating the relation between unexpected earnings and abnormal returns, especially, the change of earnings and annual abnormal returns. This line of research has been relying on simple arguments: that the unexpected earnings determine the 'theoretically correct' concept in explaining the unexpected returns (Ohlson and Shroff, 1992), and that the change in annual earnings approximates unexpected earnings relating to the annual unexpected returns because annual earnings follow a random walk process. The unexpected returns are the differences between the actual returns and expected returns. In many of the earnings-returns, the expected returns are derived based on the CAPM (Capital Asset Pricing Model) theory, i.e., the expected returns include two elements: the risk-free returns and the risk premium. The risk premium can be expressed in terms of market returns multiplied by the market beta which is measured as the covariance of the individual returns and market returns. However, to include the expected rate of return in the theoretical abnormal returns model, the expected rate of return does not need to be explicitly expressed using the CAPM model expression (an example of this explicit expression is given in Christie, 1987). The two-period model described in Watts and Zimmerman (1986) can be applied to the general model used in this paper.

The expected rate of returns for future periods at the end of period t-1 can be expressed as $\hat{r}_{it-1}$, the expected returns for period t are then:

$$P_{it-1} \cdot \hat{r}_{it-1} = \frac{\hat{D}_{it-1}}{\hat{r}_{it-1}} \cdot \hat{r}_{it-1} = \hat{D}_{it-1} \equiv \hat{E}_{it-1}. \tag{22}$$

By subtracting $\hat{D}_{it-1}$ from the left-hand side of the equation and subtracting $\hat{E}_{it-1}$ from the right-hand side, the unexpected returns for the returns equations (19) to (21) can be restated as:

$$\frac{P_{it} - P_{it-1} + D_{it} - \hat{D}_{it-1}}{P_{it-1}} = \frac{1}{\hat{r}_{it}} \cdot \frac{\hat{E}_{it}}{P_{it-1}} + \frac{\Delta \hat{E}_{it}}{P_{it-1}} - 1, \tag{23}$$

$$\frac{P_{it} - P_{it-1} + D_{it} - \hat{D}_{it-1}}{P_{it-1}} = \frac{1 + \hat{r}_{it}}{\hat{r}_{it}} \cdot \frac{(\Delta \hat{E}_{it})}{P_{it-1}}, \tag{24}$$

$$\frac{P_{it} - P_{it-1} + D_{it} - \hat{D}_{it-1}}{P_{it-1}} = k \cdot \frac{1}{\hat{r}_{it}} \cdot \frac{\hat{E}_{it}}{P_{it-1}} - k + (1 - k) \cdot \frac{1 + \hat{r}_{it}}{\hat{r}_{it}} \cdot \frac{(\Delta \hat{E}_{it})}{P_{it-1}}. \tag{25}$$

The equations developed so far have very strict assumptions including a) equation (1) is true, i.e., price can be fully explained by the expected dividends and the expected cost of capital, or equivalently, there are no uncertainties about future dividends and discount rates, and that the market is rational; b) the expected dividends equal to expected earnings for all the future periods (or alternatively, existence of a hundred percent payout ratio or assumption of dividend irrelevance to price); c) the expected discount rate at the end of period t and period t − 1 is the same as all future periods; d) the expected discount rates at the end of period t for all future periods are constant and e) the expected earnings (or dividends) at the end of period t equal the earnings (or dividends) in period t.[4] If any of these assumptions is violated, the

[4]This assumption is only needed for the abnormal returns model defined above.

model needs to be modified by reconstructing the model terms, by adding an error term to the left or right hand side of the equation, or by adding error terms to the variables, especially the earnings variables. From the above analysis, one can see that by relaxing any of the assumptions, the model is being complicated to various extents. However, the simple model provided here can facilitate understanding and development of more complicated models. Various accounting papers focus on evaluating the effects of the time-series behavior of earnings on the coefficient that relates to returns. I will discuss a few papers below to illustrate the use of this theoretical framework.

4. Examples

Several earnings-returns studies will be discussed here and linked to the framework provided above.

The Beaver, Lambert and Morse (1980) Study

Beaver, Lambert and Morse are silent about the theoretical value of the coefficient relating the earnings and market values; they focus on the time-series property of the earnings behavior. They start by assuming that the price is an unknown multiple of expected future earnings. They also assume that the earnings (which include an ungarbled or true earnings[5] x_t and an error term which is caused by noise) follow an ARIMA (0,1,1), or equivalently, an IMA (1,1) process. Similar to expected earnings, the true or the ungarbled earnings are unobservable; however, the ungarbled earnings are not the expected earnings referred

[5]The concept of ungarbled earnings (the true earnings) is similar to the concept of permanent earnings; however, they may differ in the inclusion of transitory earnings. The permanent earnings are usually refer to as earnings that have long-term permanent effects on returns and the transitory earnings are true earnings but only have a one-time or short-term effects on returns.

in the above framework. Their differences rely on the time-series property of earnings. I will use the BLM study to illustrate this point. The earnings error term (or noise) will be ignored in the analysis. Let E represents the true (or ungarbled) earnings and ignore the subscript i, the ARIMA (0,1,1) process implies:

$$E_t = E_{t-1} + a_t - \theta a_{t-1}, \tag{26}$$

where a_t and a_{t-1} follow a white noise series. Accordingly, the expected earnings at the end of period t-1 can be expressed as:

$$\widehat{E}_{t-1} = E_{t-1} - \theta a_{t-1}, \tag{27}$$

and the expected earnings at the end of period t is:

$$\widehat{E}_t = E_t - \theta a_t. \tag{28}$$

Substitute equation (22) into (24) and subtract equation (24) by (23), the change of the expected earnings can be expressed as:

$$\begin{aligned} \widehat{E}_t - \widehat{E}_{t-1} &= E_{t-1} + a_t - \theta a_{t-1} - \theta a_t - E_{t-1} - \theta a_{t-1} \\ &= (1-\theta)a_t. \end{aligned} \tag{29}$$

BLM use the returns model described in equation (13). The right-hand side of equation (13) can be expressed as:

$$\begin{aligned} \frac{\Delta \widehat{E}_{it}}{\widehat{E}_{t-1}} &= \frac{(1-\theta)a_t}{E_{t-1} - \theta a_{t-1}} = \frac{(1-\theta)(E_t - E_{t-1} + \theta a_{t-1})}{E_{t-1} - \theta a_{t-1}} \\ &= \frac{(1-\theta)\Delta E_t + (1-\theta)\theta a_{t-1}}{E_{t-1} - \theta a_{t-1}}. \end{aligned} \tag{30}$$

Equation (30) is similar to equation (7a) in BLM without the earnings error term. If earnings[6] follow the random walk process, i.e. $\theta = 0$, then returns can be expressed exactly as the ratio between the change of earnings over the previous period earnings. Note that, the equation (7b) in BLM assumes zero error term for the observed earnings. The equations (7b) in BLM is restated here as a reference for the discussions.

(7b) in the BLM: assuming $\theta = 0$, i.e., the earnings follow the random walk process, the expected earnings at the end of a period equal that period's earnings, and that the change of the expected earnings equals to the change of the earnings.

$$\frac{P_t - P_{t-1}}{P_{t-1}} = \frac{\Delta \widehat{E}_t}{\widehat{E}_{t-1}} = \frac{\Delta E_t}{E_{t-1}} . \tag{31}$$

The usage of prior period earnings as the scaling factor by BLM and by most of the early returns-earnings studies has problems due to the unpredictable behavior of the measurement errors contained in the denominator. After Christie (1987) suggests using the prior end-of-period price to scale the unexpected earnings, most recent studies use such price as the scaling factor instead of the prior-period reported earnings. Note that the theoretical models using either the prior-period price or the prior-period expected earnings as the scaling factor are alike; however, the prior period price has less measurement error then the reported earnings. This measurement error argument also supports the prior end-of-period price being the scaling factor as suggested by Christie.

[6]The earnings terms used in these section are referred to as the ungarbled or true earnings without measurement error. This paper focuses on the theoretical model; it is assumed that either we can observe the true earnings or the reported earnings are the true earnings. Earnings measurment errors introduce more complexity into the modeling process, which is beyond the scope of this paper.

The ERC Studies

Kormendi and Lipe (1987), Easton and Zmijewski (1989) and Collins and Kothari (1989) represent a set of rigorous research studies that investigate the empirical relationship of unexpected returns and unexpected earnings based on an explicit theoretical model. Kormendi and Lipe introduce the concept of earnings persistence, which is typically measured from ARIMA time series earnings process. Recalled the IMA (1,1) process used by BLM, the change of expected earnings is expressed as $(1-\theta)a_t$ (equation 29), where $a_t = E_t - \widehat{E}_{t-1}$, and θ is the moving average process parameter. Thus, revisions in earnings expectations have a persistence factor of $(1-\theta)$ in relation to the shock the new earnings (E_t) bring in. Note that a_t is the shock from reported ungarbled earnings (if observable) which is different from the change of expected earnings, i.e., $\widehat{E}_t - \widehat{E}_{t-1}$. In this context, the shock, a_t, is typically referred to as the unexpected earnings. In other words, when the earnings follow the ARIMA (0,1,1) process, the change of expected earnings equals $(1-\theta)$ times of the unexpected earnings, i.e.,:

$$\begin{aligned} \Delta\widehat{E}_t = \widehat{E}_t - \widehat{E}_{t-1} &= (1-\theta)UE_t \\ &= (1-\theta)(E_t - \widehat{E}_{t-1}) \\ &= (1-\theta)a_t. \end{aligned} \tag{32}$$

To link the unexpected earnings a_t to unexpected returns, we can not substitute equation (32) directly into equation (24) because equation (24) is true only when the expected earnings $\widehat{E}_t$ (equivalently, expected dividends $\widehat{D}_t$) equal current earnings E_t (or equivalently, dividends D_t), which is only true when earnings follow a random walk process. For the ARIMA (0,1,1) model, the expected earnings equal current earnings subtract a moving average term (equation 28). The illustration of the way we link equation (32) with the unexpected returns model in equation (24) shows how important it is for us to consider the assumptions used to derive the earnings-returns model. If a assumption is violated, the derived model is then not directly applicable. To clarify my point, I will demonstrate an incorrect way of deriving the unexpected earnings-unexpected returns theoretical model.

Assuming that a researcher believes that the unexpected return is a multiple of change of unexpected earnings; the multiple is *(1 +expected discount rate)/expected discount rate* as suggested Ohlson (1991). This model is described in equation (24). The researcher assumes that the earnings follow an ARIMA (0,1,1) process so that the change of expected earnings can be expressed as equation (29). A strict substitution of (29) into (24) will yield:

$$\frac{P_t - P_{t-1}}{P_{t-1}} = \frac{1+\hat{r}_t}{\hat{r}_t} \Delta E_t = \frac{1+\hat{r}_t}{\hat{r}_t}(1-\theta)a_t. \tag{33}$$

According to Kormendi and Lipe's (1987) and Collins and Kothari's (1989) definition of the ERC, it will be erroneously stated as:

$$\frac{1+\hat{r}_t}{\hat{r}_t}(1-\theta) = (1-\theta) + \frac{1+\theta}{\hat{r}_t}. \tag{34}$$

The correct way to link a_t to unexpected returns is to start from the returns model in equation (12) that:

$$\frac{P_t - P_{t-1}}{P_{t-1}} = \frac{1}{\hat{r}_t} \Delta E_t = \frac{1}{\hat{r}_t}(1-\theta)a_t. \tag{35}$$

Because $D_t - \hat{D}_{t-1} = a_t$ (based on the isomorphic assumption of dividends and earnings), equation (35) then becomes:

$$\frac{P_t - P_{t-1} + D_t - \hat{D}_{t-1}}{P_{t-1}} = (1 + \frac{1-\theta}{\hat{r}_t})a_t. \tag{36}$$

Collins and Kothari (1989) provide a comprehensive formula for estimating the relation between earnings and returns under different ARIMA earnings processes. The ERC described in (36) exactly equals to the one described by them.

The ERC's have different forms with respect to different time-series properties, the method illustrated above in searching the ERC for the ARIMA (0,1,1) process does not apply to other processes. Recall that in the equity model section, the last assumption added to obtain the simplest model form in equation (9) is the constant expected future earnings. The returns model derived from equation (9) can not be modified by simply substituting ΔE_{it} with a function of a_t if this assumption is not true. In the ARIMA (0,1,1) case, the assumption of constant future expected earnings is true and the simple substitution as illustrated above is applicable. However, if the expected earnings are not constant across all future periods, then a more complicated model as illustrated in Model 2 in equation (4) should be the starting point to derive the underlying earnings-returns relation. One of the reasons that the moving average model is more popular than the autoregressive model in the ERC studies perhaps is because the convenient proposition that the expected earnings from the moving average model are constant across all future periods. For readers who are interested in the explicit modeling of ERC using different earnings time-series process, Collins and Kothari (1989) provide an excellent general guideline in their table 1.

The Easton-Harris (1991) and Ali-Zarowin (1992) Studies

Easton and Harris (1991) combine the concept of book value measuring equity with error into the earnings valuation approach for equity valuation and derive a returns model that include both the earnings level and earnings change variables. Their model is similar to the model described in equation (21) or (25) except they are silent on the coefficient values in the returns model. Ali and Zarowin (1992) assume that the earnings follow the ARIMA (0,1,1) process, hence, the unexpected earnings are affected by both the level of current earnings and change of earnings. Their equation (3) can be easily seen from our equation

(26):

$$
\begin{aligned}
a_t &= E_t - E_{t-1} + \theta a_{t-1} \\
&= E_t - E_{t-1} + \theta(E_{t-1} - E_{t-2} + \theta a_{t-2}) \\
&= E_t - (1-\theta)E_{t-1} - \theta(1-\theta)E_{t-2} - \dots \\
&\equiv E_t - (1-\theta)(E_t - \Delta E_t) \\
&= \theta E_t + (1-\theta)\Delta E_t.
\end{aligned} \tag{37}
$$

To generate the simple relationship between the unexpected earnings and the earnings level and earning change variables, the information beyond lag 1 is all truncated. The results indicate that the unexpected earnings are weighted averages of current earnings and earnings changes. The magnitude of the moving average term θ determines the importance of the earnings level variable. The conclusion from Ali and Zarowin regarding the role of earnings level is at best a partial approximation not only because the relation between unexpected earnings and the earnings variables is simplified, but also because Ali and Zarowin rely only on an empirical specification without considering the explicit theoretical relation between earnings and abnormal returns when linking the unexpected earnings to the abnormal returns.

5. Concluding Remarks

The paper starts by using a general equity valuation model which is simplified with various assumptions and the simplest equity valuation model is used to derive the returns model. For the pedagogical purpose, the mathematical derivations are very detailed and may be even tedious; however, they illustrate the importance of 'careful thinking' in a modeling process. The models illustrated in this paper can be further complicated by relaxing several assumptions. One is the isomorphic assumptions of earnings and dividend; Ohlson (1991) considers the violation of this assumption. Another important consideration that can be further explored is the measurement errors contained in the reported earnings which require sophisticated econometric modeling techniques.

The empirical earnings-returns relation has been studied for more than two decades. While numbers of the empirical studies are overwhelming, studies focusing on the theoretical explicit relation between earnings and returns are scarce. It is not until recently that the theoretical models are used to find the important role of the earnings level variable in explaining returns. While we all know that earnings have information content because they are related to returns, why is it still important to explore the explicit relation between earnings and returns? The earnings-returns relation goes beyond the information content purpose. It serves as the fundamental concept for capital market research studies that link the external market behavior to internal management decisions, specially, the decision results that are shown through the accounting data. This paper provides a simple theoretical framework for the fundamental concept which will serve as a basis for exploring the complex relationship between market reactions and accounting information.

References

Ahmed, A.. 1994. Accounting earnings and future economic rents: An empirical analysis. *Journal of Accounting and Economics* 17 (May, 1994): 378-401.

Ali, A.. 1994. The incremental information content of earnings, Working capital from operations, and cash flows. *Journal of Accounting Research* 32 (Spring, 1994): 61-74.

Ali, A.. and Paul Zarowin. 1992. The role of earnings levels in annual earnings-returns studies. *Journal of Accounting Research* 30 (Autumn, 1992): 286-296.

Ball, R. and P. Brown. 1968. An empirical evaluation of accounting income numbers. *Journal of Accounting Research* 6 (Autumn, 1968): 159-178.

Beaver, W., R. Clark and R. Wright. 1979. The association between unsystematic security returns and the magnitude of earnings forecast errors. *Journal of Accounting Research* 17 (Autumn, 1979): 316-340.

Beaver, W., R. Lambert and D. Morse. 1980. The information content of security prices. *Journal of Accounting and Economics* 2 (1980): 3-28.

Collins, D. W. and S. P. Kothari. 1989. An analysis of intertemporal and cross-sectional determinants of earnings response coefficients. *Journal of Accounting and Economics* 11 (1989): 143-182.

Christie, A.. 1987. On cross-sectional analysis in accounting research. *Journal of Accounting and Economics* 9 (1987): 231-258.

Dhaliwal, D.S. and Reynolds, S.S.. 1994. The effect of the default risk of debt on the earnings response coefficient. *The Accounting Review* 69 (April, 1994): 412-424.

Easton, P. D. and Zmijewski, E.. 1989. Cross-sectional variation in the stock market response to accounting earning announcements. *Journal of Accounting and Economics* 11 (1989): 117-141.

Easton, P.D. and T.S. Harris. 1991. Earnings as an explanatory variable for returns. *Journal of Accounting Research* 29 (Spring, 1991): 19-37.

Fama, E.F.. 1977. Risk adjusted discount rates and capital budgeting under uncertainty. *Journal of Financial Economics* 5 (1977): 3-24.

Fama, E.F., and M. Miller. 1972. *The theory of finance*. Dryden Press, Hinsdale, IL, 1972.

Kallapur, S.. 1994. Dividend payout ratios as determinants of earnings response coefficients: A test of the free cash flow theory. *Journal of Accounting and Economics* 17 (May, 1994): 359-377.

Kormendi, R. and Lipe, R.. 1987. Earnings innovation, earnings persistence, and stock returns. *Journal of Business* (1987): 323-345.

Lev, B.. 1989. On the usefulness of earnings and earnings research: Lessons and directions from two decades of empirical research. *Journal of Accounting Research* 27 (1989): 153-202.

Lipe, R.. 1990. The relation between stock returns and accounting earnings given alternative information. *The Accounting Review* 65 (Jan. 1990): 49-71.

Miller, M. and Modigliani F. 34(4). Dividend policy, growth and the valuation of shares. *Journal of Business* (1961): 411-433.

Ohlson, J.. 1989. Accounting earnings, book value, and dividends: The theory of the clean surplus equation (Part I). *Working Paper.*

Ohlson, J.. 1991. Earnings, book value and dividends in security valuation. *Working Paper*, Columbia university, 1991.

Ohlson, J.. 1991. A synthesis of security valuation theory and the role of dividends, cash flows and earnings. *Contemporary Accounting Research* (Fall, 1991): 1-19.

Ohlson, J. and P. Shroff.. 1992. Changes vs. levels in earnings and explanatory variables for returns: Some theoretical considerations. *Journal of Accounting Research* 30 20 (Autumn, 1992): 210-226.

Ohlson, J.A.. 1983. Price earnings ratios and earnings capitalization under uncertainty. *Journal of Accounting Research* 21 (1983): 141-154.

Ohlson, J.A. , and S. Penman. 1992. Disaggregated accounting data as explanatory variables for returns. *Journal of Accounting Auditing and Finance* (1992): 553-573.

Warfield, T.D. and Wild, J.J.. 1992. Accounting recognition and the relevance of earnings as an explanatory variable for returns. *The Accounting Review* 67 (October, 1992): 821-842.

Watts, R. L. and J. L. Zimmerman. 1986. Positive accounting theory. Prentice-hall.

Wilson, G.P.. 1986. The relative information content of accruals and cash flows: Combined evidences of the earnings announcement and annual report release date. *Journal of Accounting Research*, 14 (Supplement 14, 1986): 165-200.