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產出率變異性之成本

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摘要

近年來，針對製造環境中品質管制之研究著重於探討管理生產產出率之變異性之重要性。然而傳統管理會計的探討尚未體認生產不確定性對諸如邊際貢獻分析等基本模式之影響。因此，品質管理方法一直未被納入會計成本之中，因而造成對生產產出率變異性之懲罰。

本研究藉延伸傳統邊際貢獻分析至隨機製造環境中，探討產出率變異性之影響。結果顯示每單位需求之最適預期邊際貢獻與需求無關，然而卻會隨著生產產出率變異性下降而增加。若管理會計系統中存貨以高於實際經濟殘值之成本評價，則經理人員將無法得知產出率變異性之影響。

關鍵詞：產出率，隨機製造環境，不確定性，邊際貢獻。

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INVITED EDITORIAL

Costs of Yield Variability

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Abstract

Recent research on quality control in manufacturing environments emphasizes the importance of managing the variability in production yield rates. Conventional approaches in managerial accounting, however, have generally not recognized the impact of production uncertainties on even the basic method of contribution margin analysis. As a result, quality management methods have adopted non-accounting cost heuristics that penalize variability in production yield.

In this paper we study the effect of yield variability by extending the conventional contribution margin analysis to stochastic manufacturing environments. We show that the *optimal* expected contribution margin per unit demand is independent of demand, but increases with reduction in production yield variability. If the management accounting system values inventory at cost that exceeds its true economic salvage value then the impact of yield variability will not be perceived at all by the managers.

Key words: Production yield, Stochastic manufacturing environments, Uncertainties, Contribution margin.

1. Introduction

Considerable attention has been paid in recent manufacturing management research and practice on the uncertainty inherent in production yield rates. It is generally recognized in these literature's that variability in production yield is undesirable, and should be reduced or eliminated. "Zero defects" is often presented as a goal worth striving for, even if it is rarely attainable in practice. Simple financial performance measures are perceived to reflect only the value of improving average production yield, but not the value of reducing yield variability. As a result, heuristics have been developed to penalize large variations from the production norm.[1] Nonetheless, managers find it difficult to justify investments that control yield variability on the basis of usual financial criteria. Highlighting this inadequacy of financial measures, Kaplan (1983) has called on management accountants to develop new measures of manufacturing performance.

In this paper, we extend the basic management accounting methodology of contribution margin analysis to address the case of variable production yield. In particular, we examine the optimal production input decision by the firm under production uncertainty, and derive the relation between *expected* profit and demand volume. As in the familiar certainty case, the expected profit is *linear* in demand, except that expected contribution per unit demand is smaller in the case of uncertain yield. Consequently, the impact of yield variability is captured fully in the relative expected contribution margins.

In our model, yield variability is costly because a higher than expected realized yield results in surplus output that may need to be disposed of through sec-

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[1]The quadratic loss function employed in Taguchi's method of quality control, for instance, assumes that the total societal loss due to variation from the production norm is proportional to the square of the deviation. Our focus, in contrast, is on economic loss to the firm due to variation in proportion of rejects.

ondary distribution channels at a considerably lower sales price or inventoried for possible sale in a subsequent period. A lower than expected yield may imply foregone sales as production did not meet demand, or in many organizations it may imply additional costs of confusion and congestion as expeditious steps are required to make up the shortfall in meeting the customer order.

The problem of variable production yield in chemical, metallurgical, and other process industries has long been recognized in the manufacturing management literature. More recently, the problem of highly variable yields encountered in the fabrication of high-tech components, including wafer fabrication and microelectronic assembly, has received considerable attention. Lee and Yanos (1988), for instance, report yield rates varying between 0.76 and 0.96 in a light emitting diode (LED) fabrication facility. Sepehri et al. (1986) report yield variations of 50 to 95 percent in Silicon Valley companies. Several studies, ranging from Giffler (1960) and Levitan (1960) to Porteus (1986), Lee and Rosenblatt (1986) and Sepehri et al. (1986) have considered the problems of production control and optimal lot sizing in single-stage systems where "production yield is highly variable".[2] In a similar vein we consider production decisions optimizing the trade-off between shortages and overages. This assumption of rational behavior by the decision maker is important as it provides the basis for our principal result that the optimal expected profit under uncertain yield is linear in demand.

The remainder of this paper has the following structure. Section 2 describes our basic model under uncertain yield and solves for the optimal input level as the expected profit is strictly concave in the choice of input level. Section 3 presents our principal results that the optimal expected profit is linear in demand

[2]More complex multi-stage and other manufacturing process models also exist in this literature, but their emphasis is on optimal production scheduling rather than on accounting implications of yield variability.

and the optimal expected contribution margin declines with yield variability. Section 4 concludes the paper.

2. Basic Model

We develop a single-period, single-product model for an expected profit maximizing firm. In most manufacturing establishments, particularly in plants using JIT-type pull systems, production is against customer orders. Demand (in units) is therefore known when the firm decides on the production level (x units) for the period.[3] The production level decision can be thought of as a batch size decision where the entire batch is processed through several stages. The batch size can not be augmented, except at a higher unit cost (p+t) discussed later.

The only source of uncertainty remaining after the firm chooses the production level is the variability in the production yield rate (y). This is consistent with the models in the operations management literature dealing with yield variability. Yield rate is usually not known when the production input level is chosen because of imperfections in the production process and unobserved variations in input quality, or because some of the process parameter settings are not known or controlled precisely. The yield rate y is a random variable in the interval [0, 1], distributed with a mean $\bar{y}$, probability density function $\phi(y)$ and cumulative density function $\Phi(y)$. The partial expectation function $\psi(\gamma)$ is defined as $\int_0^{\gamma} y\,\phi(y)\,dy$ for $0 \le \gamma \le 1$, with $\psi(1) = \bar{y}$. We assume $\phi(y) > 0$ on the interval $[y_L, y_H] \subseteq [0,1]$ and 0 otherwise. The inverse function $\psi^{-1}(y)$ is defined over $[0, \bar{y}]$ with $\psi^{-1}(0)$ and $\psi^{-1}(\bar{y})$ defined to be y_L and y_H respectively.

Price ($p per unit), production costs (variable: $v per unit of input; fixed: $f for the period) and demand (m units) are known with certainty. The production

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[3]Taiichi Ohnu, the father of the Toyota production system advocates making what the customer wants, when he wants it.

process yields xy good units, and the remaining (1-y)x units are scrapped, obtaining a price of \$s per unit of scrap. If the number of good units (xy) exceeds the demand (m) then the surplus units (= xy - m) may be sold through secondary distribution channels at a salvage price \$g per unit ($g<p$). Alternatively, if such surplus inventory is to be sold in a subsequent period, then g can be thought of as the discounted expected net production cost (after scrap credit) in that period less the discounted expected storage and handling cost per unit.

If the number of good units is less than the demand then the unsatiated demand may involve a cost of \$t (≥0) per unit due to loss of goodwill with customers. Alternatively, \$(p+t) may be regarded as the higher variable cost per unit of good production for the completion of the remainder of the customer order. These costs are higher because rework or other means for expeditious production of the unfilled part of the customer order create confusion and congestion on the shop floor and require considerable additional effort to move and reschedule other jobs so that the present order can be completed.

The *ex post* profit function π(x,y) can now be written as

$$(1)\quad \pi(x,y;m)=\begin{cases} pxy+sx(1-y)-t(m-xy)-vx-f & \text{if } xy\le m \\ pm+sx(1-y)+g(xy-m)-vx-f & \text{if } xy\ge m \end{cases}.$$

Therefore,

$$(2)\quad \frac{\partial\pi(x,y;m)}{\partial x}=\begin{cases} (p+t)y+s(1-y)-v & \text{if } x<m/y \\ gy+s(1-y)-v & \text{if } x>m/y \end{cases},$$

and

$$(3)\quad \frac{\partial\pi(x,y;m)}{\partial y}=\begin{cases} (p+t-s)x & \text{if } y<m/x \\ (g-s)x & \text{if } y>m/x \end{cases}.$$

Since by assumption p>g and t≥0, the profit π(x,y; m) as a function of x given y, or as a function of y given x, is evidently concave and piecewise linear with a kink at xy=m. See Figures 1 and 2.

Figure 1
Profit π (x, y; m) as a function of input x given yield y and demand m

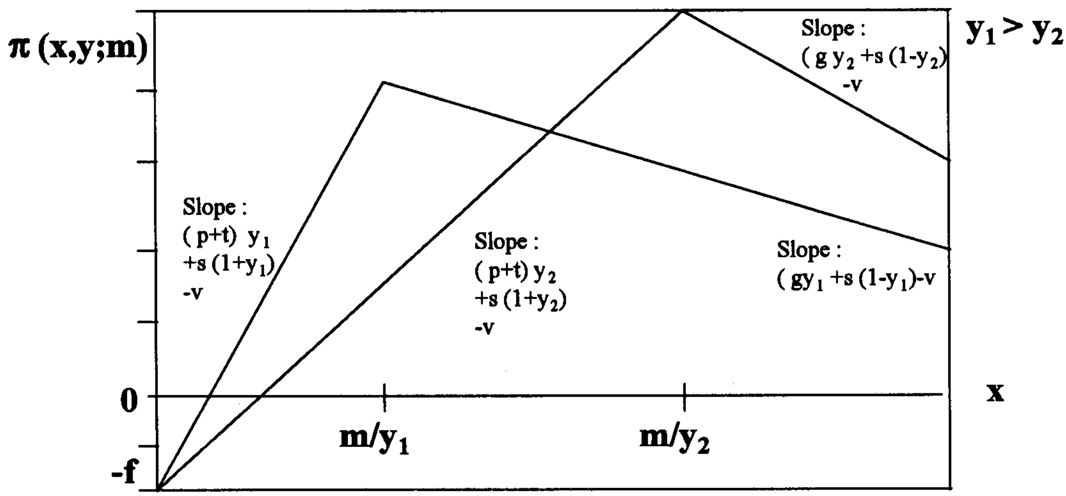
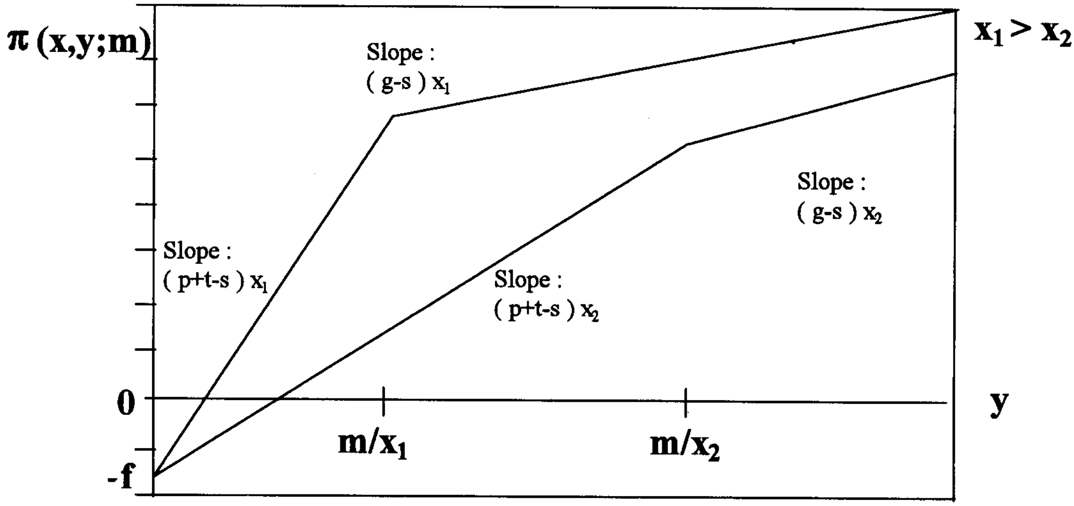


Figure 2
Profit π (x, y; m) as a function of yield y given input x and demand m



The expected profit $E_y[\pi(x,y;m)]$ under uncertain yield as a function of input level x can next be written as:

$$(4)\ E_y[\pi(x,y;m)] = p\int_0^{m/x} xy\phi(y)dy + p\int_{m/x}^{1} m\phi(y)dy + sx(1-\bar{y}) + g\int_{m/x}^{1}(xy-m)\phi(y)dy - t\int_0^{m/x}(m-xy)\phi(y)dy - vx - f.$$

The first two terms in the above expression reflect the expected value of sales revenue, the third represents the expected scrap revenue, and the fourth the expected salvage value of surplus inventory. The fifth term reflects the cost of unsatiated customer orders and the last two terms represent the variable and fixed costs of production for the input quantity. Equation (4) can also be expressed as:

(4A') $E_y[\pi(x,y;m)] = (p+t-g)[x\psi(m/x) - m\Phi(m/x)] + (p-g)m + gx\bar{y} + sx(1-\bar{y}) - vx - f.$

Before analyzing this general expression for expected profit under uncertain yield, we first consider as a benchmark the case when the yield rate $\bar{y}$ is known with certainty. In both the certainty and the uncertainty cases, the (expected) unit variable cost of good output, after due credit for scrap, is $c = [v - s(1-\bar{y})]/\bar{y}$. If it is less than the salvage price (g), the firm will be motivated to choose the maximum level of production input (limited by plant capacity) regardless of demand level m. On the other hand, if the (expected) unit variable cost of good production is greater than the sales price (p) of good output, then the firm will be motivated to close the (plant) facility regardless of the demand level. Therefore, to exclude these extreme situations when the expected profit is independent of demand, we bound the (expected) variable cost of production between g and p; i.e.,

(5) $g < [v - s(1-\bar{y})]/\bar{y} < p.$

This assumption will be maintained throughout the paper.

A number that is important in our subsequent analysis is the "critical fractile" $\lambda = \lambda_1 / (\lambda_1 + \lambda_2)$ where $\lambda_1 = [v - s(1-\bar{y})]/\bar{y} - g = c - g$ is the surplus inventory mark-down, and $\lambda_2 = p - [v - s(1-\bar{y})/\bar{y} = p - c$ is the sales mark-up. Evidently, since $\lambda_1, \lambda_2 > 0$, we have $0 < \lambda < 1$. We next present an expression for the optimal profit under certainty in a form that will be useful for comparison later with the uncertainty case.

Lemma 1: The optimal profit $\pi^{c*}(m)$ for a known demand m, when the yield rate is known with certainty is given by

(6) $\pi^{c*}(m) = m(p-g)(1-\lambda) - f.$

Proof of Lemma 1: If the known production yield rate is $\bar{y}$, then assumption (5) implies that the optimal production input quantity $x^{C*}(m) = m/\bar{y}$. The optimal profit then is

$$\begin{aligned}\pi^{c*}(m) &= pm - [v - s(1-\bar{y})]m/\bar{y} - f \\ &= m\lambda_2 - f\end{aligned}$$

Now, $$\begin{aligned}1-\lambda &= \lambda_2/(\lambda_1+\lambda_2) \\ &= \lambda_2/(p-g)\end{aligned}$$

.Therefore, $\pi^{c*}(m) = m(p-g)(1-\lambda) - f$ Q.E.D.

Before moving on to our principal results in the next section, we summarize below in Table 1 the principal notation that we employ in this paper.

Table 1
Notation

Demand	m units
Production input	x units
Yield rate	y
Fixed production cost for the period	\$ f
Variable cost per unit	\$ v
Sales price per unit	\$ p
Salvage value per unit	\$ g
Scrap value per unit	\$ s
Cost per unit of unsatiated demand	\$ t
Profit	\$ π
Probability density function	$\phi(y)$
Cumulative density function	$\Phi(y)$
Partial expectation function	$\psi(y)=\int_0^y \tilde{y}\phi(\tilde{y})d\tilde{y}$
Expected yield	$\bar{y}$
Expected variable cost	$c \equiv \left[v - s(1-\bar{y})\right]/\bar{y}$
Surplus inventory mark-down	$\lambda_1 \equiv \left[v-s(1-\bar{y})\right]/\bar{y}-g=c-g$
Sales mark-up (equals optimal contribution margin per unit demand under uncertainty)	$\lambda_2 \equiv p-\left[v-s(1-\bar{y})\right]/\bar{y}=p-c$
Critical fractile under certainty	$\lambda \equiv \lambda_1/(\lambda_1+\lambda_2)$
Critical fractile under uncertainty	$\mu \equiv \Phi(\psi^{-1}(\lambda(p-g)/(p+t-g)))$
Optimal expected contribution margin per unit demand under uncertainty	$\eta \equiv (p-g)[1-\mu-\mu t/(p-g)]$
Ratio of optimal expected contribution margin per unit demand under uncertainty to that under certainty	$\rho \equiv [(1-\mu)/(1-\lambda)]-[\mu t/\lambda_2]$

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Ratio of optimal expected contribution margin per unit demand under uncertainty to that under certainty	$\rho \equiv [(1-\mu)/(1-\lambda)]-[\mu t/\lambda_2]$

3. Principal Results

Under yield certainty, the optimal contribution margin per unit demand is $=(p-g)(1-\lambda)=\lambda_2=p-c$, which reduces to p-v when $\bar{y}=1$ (that is, we always have 100% yield). The optimal input quantity under certainty is $x^{c*}=m/\bar{y}$, chosen to yield good output exactly equal to the known demand, see Figure 1. When the yield rate is uncertain, the cost of shortages and overages must be optimally traded off. Loosely put, at the optimal production input level, the expected cost of a marginal unit must be exactly equal to the expected benefit from it. The expected production cost is the variable cost \$v less the expected scrap credit $\$s(1-\bar{y})$. If the good production exceeds demand, then the marginal unit will be salvaged for \$g. If the good production is less than demand, then the marginal unit contributes a price \$p and saves a potential goodwill loss \$t. The probability of good production being less than demand is $\Pr\{x^*y<m\}=\Pr\{y<m/x^*\}$. Therefore, intuitively we can equate

$$(7)\qquad g\int_{m/x^*}^{1}x^*y\phi(y)dy+(p+t)\int_0^{m/x^*}x^*y\phi(y)dy=v-s(1-\bar{y}).$$

That is $(p+t-g)\psi(m/x^*)=v-s(1-\bar{y})-g\bar{y}=\lambda(p-g)\bar{y}$ or $x^*=m/\psi^{-1}(\lambda(p-g)\bar{y}/(p+t-g))$. We formalize this intuitive argument in the following lemma:

Lemma 2: When the yield rate is uncertain, the expected profit $E_y[\pi(x,y;m)]$ given a demand m is strictly concave in x, and the optimal input quantity is $x^*=m/\psi^{-1}(\xi)>m$ where $\xi=\lambda\bar{y}(p-g)/(p+t-g)$.

Proof of Lemma 2: Differentiating the expression for expected profit $E(\pi)$ in (4) with respect to x, we obtain

$$\partial E(\pi)/\partial x = (p+t)[\psi(m/x)+x(m/x)\phi(m/x)(-m/x^2)]+(p-g)$$
$$(-m)\phi(m/x)(-m/x^2)+g[(-\psi(m/x))+x(-m/x)\phi(m/x)$$
$$(-m/x^2)]-tm\phi(m/x)(-m/x^2)+s(1-\bar{y})-v$$
$$=(p+t-g)\psi(m/x)+g\bar{y}+s(1-\bar{y})-v$$

and $\partial^2 E(\pi)/\partial x^2 = (p+t-g)(m/x)\phi(m/x)(-m/x^2) < 0.$

Therefore, the first order condition for optimization, which requires setting $\partial E(\pi)/\partial x = 0$, yields

$$\psi(m/x^*) = [v-s(1-\bar{y})-g\bar{y}]/[p+t-g]$$
$$=\lambda\bar{y}(p-g)/(p+t-g)$$

Solving for x^* gives

$x^* = m/\psi^{-1}(\xi)$ where

$$\xi = \frac{\lambda\bar{y}(p-g)}{(p+t-g)} = \frac{(p-c)\bar{y}}{(p+t-g)}$$

Now, since $\lambda < 1$ and $(p-g) \le (p+t-g)$, we have $\xi < \bar{y}$. Therefore, $\psi^{-1}(\xi) < \psi^{-1}(\bar{y}) = 1$, and hence $x^*(m) = m/\psi^{-1}(\xi) > m$. Q.E.D.

Having determined the optimal input quantity $x^*(m)$ we can now compute the optimal expected profit when yield rate is uncertain, by substituting in the value of $x^*(m)$ in the expression for $E(\pi)$ given by (4'). For notational convenience, we write $\mu = \Phi(\psi^{-1}(\xi))$. The principal results obtained thus are summarized in the following:

Proposition 1: When the yield rate is uncertain,

(a) the optimal expected profit is given by the expression

(8) $$E[\pi^*(m)] \equiv E_y[\pi(x^*(m),y;m)] = m(p-g)[1-\mu-\mu t/(p-g)]-f.$$

and thus, $E[\pi^*(m)]$ is linear in the demand m, and

(b) the optimal expected profit under uncertainty is less than the optimal profit under certainty, that is $E[\pi^*(m)] < \pi^{c*}(m)$.

Proof of Proposition 1:

(a) Substituting the expression for optimal $x^*(m)$ in equation (4'), we obtain

$$\begin{aligned} E_y[\pi(x^*(m),y;m)] &= (p+t-g)(x^*\xi - m\mu) + (p-g)m + gx^* + sx^*(1-\bar{y}) - vx^* - f \\ &= (p+t-g)x^*\xi + m(p-g)(1-\mu-\mu t/(p-g)) - x^*[v-s(1-\bar{y})-g\bar{y}] - f \\ &= \lambda\bar{y}(p-g)x^* + m(p-g)(1-\mu-\mu t/(p-g)) - \lambda\bar{y}(p-g)x^* - f \\ &= m(p-g)(1-\mu-\mu t/(p-g)) - f. \end{aligned}$$

Note that ξ and hence μ are both independent of m. Therefore, $E[\pi^*(m)]$ is linear in m.

(b) Since $\pi(x,y,m)$ is strictly concave in y, given x and m, it follows from Jensen's inequality that

$$\pi(x,;m) > E_y[\pi(x,y;m)] \quad \forall x \text{ and } m.$$

In particular for the optimal $x^*(m)$ under uncertainty, we have

$$\pi(x^*(m),\bar{y};m) > E_y[(x^*(m),\bar{y};m)] = E[\pi^*(m)] \ \forall m.$$

Therefore, $\pi^{c*}(m) \geq \pi(x^*(m),\bar{y};m) > E[\pi^*(m)]$.

Let $\eta \equiv (p-g)[1-\mu-\mu t/(p-g)]$ denote the optimal expected contribution margin per unit demand under uncertain yield. On comparing (8) and (6) it is evident that the ratio of the optimal expected contribution margins per unit demand in the uncertainty and certainty cases is given by

$$(9) \quad \rho \equiv \frac{(p-g)[1-\mu-\mu t/(p-g)]}{(p-g)(1-\lambda)} = \frac{1-\mu}{1-\lambda} - \frac{\mu t}{\lambda_2}.$$

This follows from the identity $(1-\lambda)(p-g) = \lambda_2$. Result (b) in Proposition 1 also implies that $\rho < 1$. Q.E.D.

We have considered the economic profit under uncertainty in the above results. How does it relate to the accounting profit? The actual sales of good output and scrap, and variable and fixed costs are reflected directly in the accounting system. Surplus inventory, however, is usually valued at cost (both variable and apportioned fixed production costs), and therefore exceeds our term g which is assumed to be less then $[v - s(1-\bar{y})]/\bar{y}$, the expected variable production costs after credit for scrap. The cost of unsatiated customer demand (our term t) does not appear directly in the accounting system. Therefore, the accounting profit is likely overstated relative to the economic profit in our model, and hence the true impact of the variability in the yield rate is likely understated in the short run. In the longer run, however, overvalued beginning inventory and adverse impact on customer satisfaction (and hence demand) are both likely to be reflected in the bottom line.

Since the terms g and t are typically understated (or excluded) in traditional management accounting systems, we consider next their impact on the optimal production level $x^*(m)$.

Proposition 2: When the yield rate is uncertain,

(a) (i) $\partial x^*(m) / \partial g > 0$, and (ii) $\partial x^*(m)\partial t > 0$, and

(b) (i) $\lim_{\lambda \to 0} [\rho] = 0$ and (ii) $\lim_{t \to 0} [\rho] = \dfrac{1 - \Phi(\psi^{-1}(\lambda \bar{y}))}{1 - \lambda} < 1.$

Proof of Proposition 2:

(a) (i) $$\frac{\partial \xi}{\partial g}=\frac{\partial}{\partial g}\left[\frac{(c-g)\bar{y}}{(p+t-g)}\right]$$
$$=\frac{(c-g)\bar{y}}{(p+t-g)^{2}}-\frac{\bar{y}}{(p+t-g)}$$
$$=\frac{\bar{y}}{(p+t-g)^{2}}[c-(p+t)]$$
<0 because $p>c$, by assumption.

Next, $$\frac{\partial}{\partial \xi}\chi^{*}(m)=\frac{\partial}{\partial \xi}\left[\frac{m}{\psi^{-1}(\xi)}\right]=\frac{-m\left[\frac{\partial}{\partial \xi}\psi^{-1}(\xi)\right]}{\left[\psi^{-1}(\xi)\right]^{2}}<0,$$

because $\psi^{-1}(\xi)$ is strictly increasing in ξ.

Therefore, $$\frac{\partial}{\partial t}\chi^{*}(m)=\frac{\partial}{\partial \xi}\chi^{*}(m)\bullet\frac{\partial \xi}{\partial g}>0.$$

(ii) $$\frac{\partial \xi}{\partial t}=\frac{\partial}{\partial t}\left[\frac{(c-g)\bar{y}}{(p+t-g)}\right]=-\frac{(c-g)\bar{y}^{2}}{(p+t-g)}<0,$$ and therefore

$$\frac{\partial}{\partial t}x^{*}(m)=\frac{\partial}{\partial \xi}x^{*}(m)\bullet\frac{\partial \xi}{\partial t}>0.$$

(b) (i) As $\lambda_1 \to 0$, $g \to [v-s(1-\bar{y})]$, $\xi \to 0$ and $\mu \to 0$. Therefore,

$$\lim_{\lambda\to 0}[\rho]=\lim_{\lambda\to 0}\left[\frac{1-\mu}{1-\lambda}\right]=1$$

(ii) $$\lim_{t\to 0}[\rho]=\lim_{t\to 0}\left[\frac{1-\mu}{1-\lambda}\right]=\frac{1-\Phi(\psi^{-1}(\lambda\bar{y}))}{1-\lambda}<\frac{1-\mu}{1-\lambda}.$$

We will show that this limit is less than 1 by showing that μ is greater than λ. Toward this end we begin by defining the function $\Lambda(\kappa)=\psi(\kappa)/\bar{y}$, so that

$$\Lambda(\kappa)-\Phi(\kappa)=\int_0^{\kappa} y\varphi(y)dy-\bar{y}\int_0^{\kappa}\varphi(y)dy<0 \qquad \forall\kappa\varepsilon[0,1], \text{unless } \Phi(\kappa)=0,1.$$

Next, let $u_1=\Phi^{-1}(\lambda)$ and $u_2=\psi^{-1}(\lambda\bar{y})$, so that $\lambda=\Phi(u_1)=\psi(u_2)/\bar{y}=\Lambda(u_2)$ $<\Phi(u_2)$. since Φ is monotone increasing, we have $u_1<u_2$,

$\Phi^{-1}(\lambda)<\psi^{-1}(\lambda\bar{y})$ and $\lambda<\Phi(\psi^{-1}(\lambda\bar{y}))<\mu$. Q.E.D.

How large can the impact on optimal expected contribution margin be if yield uncertainty is eliminated? The next lemma shows that at least in one special case it can be as large as 100%. This occurs for the uniform distribution when the sales mark-up is negligible in comparison with surplus inventory mark down, and yield variability is large.

Lemma 3: If the yield rate is uniformly distributed on $[\bar{y}-\sigma,\ \bar{y}+\sigma]$ and t=0 then $\lim_{\lambda\to 1}[\rho]=\bar{y}/(\bar{y}+\sigma)$, and $\inf\{\rho\}=\frac{1}{2}$.

Proof of Lemma 3:

For the uniform distribution, $\psi(m/x)=[m^2/x^2-(\bar{y}-\sigma)^2]/4\sigma$. Therefore, from Lemma 2, we have

$[m^2/x^{*2}-(\bar{y}-\sigma)^2]/4\sigma=\lambda\bar{y}$, that is $x^*=m/k$ where $k^2=4\sigma\lambda\bar{y}+(\bar{y}-\sigma)^2$

and $\mu=\Phi(k)=[k-(\bar{y}-\sigma)]/2\sigma$.

Writing $\delta=1-\lambda$, and using Taylor series expansion for $k=[(\bar{y}+\sigma)^2-4\sigma\delta\bar{y}]^{1/2}$ at $\bar{y}+\sigma$, we obtain

$$\rho=\frac{1}{\delta}\left[1-\frac{1}{2\sigma}\left[-(\bar{y}-\sigma)+(\bar{y}+\sigma)-\frac{4\sigma\delta\bar{y}}{2(\bar{y}+\sigma)}+\text{terms involving higher powers of }\delta\right]\right]$$

$$=[\bar{y}/(\bar{y}+\sigma)]+[\text{terms involving }\delta]$$

Therefore, $\lim_{\delta\to 0}[\rho]=\bar{y}/(\bar{y}+\sigma)$

Next, since $(\bar{y}-\sigma\lambda)^2>(\bar{y}-\sigma)^2$, we have $\bar{y}+\sigma\lambda>k$. This in turn implies that

$\mu = [k - \bar{y} + \sigma]/2\sigma < \sigma[1+\lambda]/2\sigma$, and hence
$\rho = [1-\mu]/[1-\lambda] > \frac{1}{2}$.

Also, $\lim_{\lambda \to 1}[\rho] = 1/$ for $\bar{y} = \sigma$ Therefore, $\inf\{\rho\} = \frac{1}{2}$ Q.E.D.

We have so far assumed $\lambda>0$, but if a firm values inventory at cost (= [v-s(1-$\bar{y}$)]/$\bar{y}$ per unit) and incorporates that same inventory valuation for our salvage value term g in its decision rule about the optimal input level, then $\lambda_1=0=\lambda$. Therefore, result (c) in Proposition 2 above suggests there will be no difference between the *perceived* profit under uncertainty and that under certainty.[4] In fact, if $\lambda=0$, there is no penalty for overages and hence there is no incentive to choose a production level below full capacity. To provide the appropriate motivational impetus and avoid preoccupation with full capacity utilization, it is important, therefore, to reflect the full carrying cost of surplus inventory and value it at below production cost for managerial purposes.

Our recommendation for accounting practice thus echoes Drucker's (1990, p. 98) call to recognize that finished goods inventory is a "sunk cost," and not an "asset," as it ties down expensive money and absorbs costly time. Our observation is also similar in spirit to Kaplan's [1983, p. 701] comment on scrap valuation: "It is customary to assign to scrap at least its raw-material value were it to be purchased or sold on the open market. Perhaps, however, the scrap value should be reduced by the increased materials handling and storage costs, plus the cost of disrupting the production schedule, when defective output is produced. This places more of a premium on eliminating defective items in the production process."

We turn next to the impact of an increase in the yield variability when the mean yield remains unchanged.

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[4]Optimal expected contribution margin under uncertainty is less than that under certainty even if we set t=0, but this distinction is lost when inventory is valued at cost (i.e., $\lambda_1 = 0$).

Proposition 3: If the yield distribution $\Phi_1(y)$ second order stochastically dominates the yield distribution $\Phi_2(y)$ with the same mean $\bar{y}$ then (a) the optimal expected profit under Φ_2 is lower than under Φ_1, and (b) the optimal production input level $x^*(m)$ is lower under Φ_2.

Proof of Proposition 3:

(a) Since $\Phi_1(y) \underset{SSD}{\geq} \Phi_2(y)$, we have

$$S(Z) \equiv \int_0^z (\Phi_1(y) - \Phi_2(y))dy \leq 0 \quad \forall z\varepsilon[0,1]$$

We will show that this implies that $E_1[\pi(x^*(m), y; m)] > E_2[\pi(x^*(m), y; m)]$.

Note that $E(\pi_1^* - \pi_2^*) = \int_0^1 \pi(y)d(\Phi_1(y) - \Phi_2(y))$.

Integrating by parts, we get

$$E(\pi_1^* - \pi_2^*) = \pi(y)[\Phi_1(y) - \Phi_2(y)]_0^1 - \int_0^1 \frac{d}{dy}\pi(y)\int d[\Phi_1(y) - \Phi_2(y)]dy$$

Since $\Phi_1(0) = \Phi_2(0) = 0$ and $\Phi_1(1) = \Phi_2(1) = 1$, the first term above is zero and the second term can be rewritten as:

$$E(\pi_1^* - \pi_2^*) = -\int_0^1 \pi(y)dS(y).$$

Integrating by parts, we get

$$E(\pi_1^* - \pi_2^*) = -[\pi(y)S(y)]_0^1 + \int_0^1 S(y)d\pi(y)$$

Next, we observe that

$$S(0) = \int_0^0 (\Phi_1(y) - \Phi_2(y))dy = 0, \text{and}$$

$$S(1) = \int_0^1 (\Phi_1(y) - \Phi_2(y))dy$$

$$= \left[(\Phi_1(y) - \Phi_2(y))y\right]_0^1 - \int_0^1 y d(\Phi_1(y) - \Phi_2(y)) = 0$$

because $E_1(y) = E_2(y) = \bar{y}$. Further, because $\pi'(y)$ is decreasing in y, we have

$$E(\pi_1^* - \pi_2^*) = \int_0^1 S(y)\, d\pi'(y) \geq 0.$$

(b) For $y \geq 0, \Phi_1(y) \underset{SSD}{\geq} \Phi_2(y) \Rightarrow \int_0^{\xi} y[\Phi_1(y) - \Phi_2(y)]dy \leq 0 \ \forall \xi \in [0,1]$

$$\Rightarrow \int_0^{\xi} y\Phi_1(y)\, dy \leq \int_0^{\xi} y\Phi_2(y)\, dy$$

$$\Rightarrow \psi_1^{-1}(\xi) \leq \Psi_2^{-1}(\xi)$$

Since $x^*(m) = m / \psi^{-1}(\xi)$, it follows that $x_1^*(m) > x_1^*(m)$. Q.E.D.

The optimal expected profit thus declines with increased yield variability. The production input level is also optimally decreased in response to increased yield variability.

4. Concluding Remarks

Variability in yield causes costs of shortages and overages in differentiated firms emphasizing customer-specific batch production. Shortage costs are the additional costs of expeditious completion of the balance of a customer order or foregone sales and loss of goodwill with customers. Overage costs result from lower salvage value for surplus inventory and from additional inventory carrying costs. Eliminating yield uncertainty can result in substantial increase in the expected contribution margin per unit demand. The impact on expected contribution margin increases with yield variability, and is high also when the accounting system values inventory well above its true net salvage value, and ignores the expected cost of production to make up any shortage.

Recognition of yield variability costs emphasizes the value of managerial efforts to understand and reduce yield variability. Improvements in the production process that reduce yield variability can be very valuable even when they do not improve the average yield rate.[5] The framework presented in this paper for contribution margin analysis under uncertain yield makes it possible to reflect the value of such process improvements. To the extent that management accounting practices reflect the underlying production economics, the value from investment of people and capital resources to improve manufacturing practices is likely to be better appreciated, especially in firms which rely on financial performance measures to manage operations.

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[5]Firms emphasizing excellence in manufacturing focus on their best performance to understand factors contributing to it. By identifying and replicating the conditions that led to the best performance, they can collapse the yield distribution towards its high end and improve both the mean and the variance.

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